

5 Rationale Zahlen

5.12 Übungen Frommenwiler

$$55. \quad b) \quad \frac{\overset{-3}{\cancel{12}} m}{\underset{1}{\cancel{4}}} - \frac{\overset{-2}{\cancel{8}} n}{\underset{1}{\cancel{4}}} + \frac{\overset{-1}{\cancel{2}}}{\underset{2}{\cancel{4}}} = \underline{\underline{-3m + 2n - \frac{1}{2}}}$$

$$d) \quad \frac{\overset{1}{\cancel{a^2}}}{\underset{1}{\cancel{a^2}}} - \frac{\overset{1}{\cancel{a}}}{\underset{1}{\cancel{a^2}}} - \frac{1}{a^2} = \underline{\underline{1 - \frac{1}{a} - \frac{1}{a^2}}}$$

$$e) \quad \frac{\overset{-2}{\cancel{8}} m^{\overset{1}{\cancel{1}}}}{\underset{1}{\cancel{4}} m^{\overset{1}{\cancel{1}}} n} + \frac{\overset{-3}{\cancel{12}} m^{\overset{1}{\cancel{1}}} \overset{1}{\cancel{1}}}{\underset{1}{\cancel{4}} m^{\overset{1}{\cancel{1}}} \overset{1}{\cancel{1}}} - \frac{\overset{-1}{\cancel{4}} n^{\overset{1}{\cancel{1}}}}{\underset{1}{\cancel{4}} m^{\overset{1}{\cancel{1}}} \overset{1}{\cancel{1}}} - \frac{\left(\overset{-1}{\cancel{4}} m^{\overset{1}{\cancel{1}}} \overset{1}{\cancel{1}} \right) \left(\overset{1}{\cancel{4}} mn \right)}{\underset{1}{\cancel{4}} m^{\overset{1}{\cancel{1}}} \overset{1}{\cancel{1}}} = \underline{\underline{-\frac{2}{n} - \frac{3}{m} + \frac{n}{m^2} + 4n}}$$

$$56. \quad a) \quad \frac{-x^3 + 6x}{7} = \underline{\underline{-\frac{1}{7}x^3 + \frac{6}{7}x}}$$

$$b) \quad \frac{2a^2 - 4a + 6}{4} = \underline{\underline{\frac{1}{2}a^2 - a + \frac{3}{2}}}$$

$$d) \quad \frac{x - \sqrt{3}x + 1.5}{3} = \frac{x(1 - \sqrt{3}) + 1.5}{3} = \underline{\underline{\left(\frac{1 - \sqrt{3}}{3} \right) x + \frac{1}{2}}}$$

$$57. \quad a) \quad (a - b) \left(\frac{b + a}{ab} \right) = \underline{\underline{\frac{a^2 - b^2}{ab}}}$$

$$b) \quad \left(\frac{m^2 - n^2}{mn} \right) \left(\frac{m^2 + n^2}{mn} \right) = \underline{\underline{\frac{m^4 - n^4}{(mn)^2}}}$$

$$c) \left[\frac{a+1}{a(a+1)} - \frac{a}{a(a+1)} \right]^2 = \left[\frac{a+1-a}{a(a+1)} \right]^2 = \left[\frac{1}{a(a+1)} \right]^2 = \underline{\underline{\frac{1}{a^2(a+1)^2}}}$$

$$\frac{\cancel{x}}{x^2+y^2} \cdot \frac{2x^2-(x-y)(x+y)}{\cancel{x}(x+y)} = \frac{2x^2-(x^2-y^2)}{(x^2+y^2)(x+y)} =$$

$$d) \frac{2x^2-x^2+y^2}{(x^2+y^2)(x+y)} = \frac{\cancel{x^2+y^2}}{(\cancel{x^2+y^2})(x+y)} = \frac{1}{\underline{\underline{x+y}}}$$

$$58. \ a) \frac{\cancel{u}}{u+1} \left(\frac{1-u^2}{\cancel{u}} \right) - \frac{\cancel{u}}{1-u} \left(\frac{1+u^2}{\cancel{u}} \right) = \frac{(1-u) \cancel{(1+u)}}{\cancel{u+1}} - \frac{1+u^2}{1-u} =$$

$$\frac{(1-u)^2 - 1 - u^2}{1-u} = \frac{1 - 2u + u^2 - 1 - u^2}{1-u} = \frac{-2u}{1-u} = \underline{\underline{\frac{2u}{u-1}}}$$

$$\frac{a(a+b)-b(a-b)}{(a-b)(a+b)} \cdot \frac{b^2-a^2}{a^2b^2} + \frac{1}{b^2} = \frac{a^2+ab-ab+b^2}{(a-b)\cancel{(a+b)}} \cdot \frac{(b-a)\cancel{(b+a)}}{a^2b^2} + \frac{1}{b^2} =$$

$$b) \frac{a^2+b^2}{\cancel{(a-b)}} \cdot \frac{\cancel{(-b+a)}}{a^2b^2} + \frac{1}{b^2} = \frac{-a^2-b^2}{a^2b^2} + \frac{a^2}{a^2b^2} = \frac{-\cancel{b^2}}{a^2\cancel{b^2}} = \frac{-1}{a^2} = \underline{\underline{-\frac{1}{a^2}}}$$

$$(a-1) \cdot \frac{a-a^2+1}{1-a} \cdot \frac{\overbrace{(a+1)^2-a^2}^{3. \text{ Binom}}}{a+1} =$$

$$c) (a-1) \cdot \frac{(a-a^2+1)\cancel{(-1)}}{(1-a)\cancel{(-1)}} \cdot \frac{\overbrace{(a+1+a)(a+1-a)}^{3. \text{ Binom angewendet}}}{a+1} =$$

$$\cancel{(a-1)} \cdot \frac{-a+a^2-1}{\cancel{a-1}} \cdot \frac{2a+1}{a+1} = \underline{\underline{\frac{(a^2-a-1)(2a+1)}{a-1}}}$$

$$\frac{1}{\cancel{x-1}_1} \cdot \frac{\cancel{(x-1)}^1 (x+1)}{\cancel{(1-x)}_1} \cdot \frac{(1-x)^2}{(1+x)^2} \cdot \frac{x^2(x-1) - (x-1)}{(x-1)^2} =$$

d)

$$\frac{(1-x)^2 \overbrace{(x^2-1)}^{\text{Binom}} \cancel{(x-1)}^1}{\cancel{(1-x)}_1 (1+x)^2 (x-1)^2} = \frac{(1-x) \cancel{(x-1)}^1 (x+1)}{(1+x)^2 \cancel{(x-1)}^1} = \underline{\underline{\frac{1-x}{1+x}}}$$

$$x \left(\frac{x^2 - 3x + 2}{x^2} \right)^2 \left[\left(1 - \frac{x}{x-2} \right) \left(1 + \frac{x}{x-2} \right) \right] = \frac{\cancel{x}^1 (x^2 - 3x + 2)^2}{x^4} \cdot \frac{x-2-x}{x-2} \cdot \frac{x-2+x}{x-2} =$$

e)

$$\frac{[(x-1)(x-2)]^2}{x^3} \cdot \frac{-2}{x-2} \cdot \frac{2x-2}{x-2} = \frac{(x-1)^2 \cancel{(x-2)}^2 (-2) \cdot 2(x-1)}{x^3 \cancel{(x-2)}^2} =$$

$$\underline{\underline{\frac{4(x-1)^3}{x^3}}} = \underbrace{\frac{4[-(-x+1)]^3}{x^3}}_{\text{Minuszeichen kann in den Term } (x-1)^3 \text{ gebracht werden}} = \frac{4[(-1)^3(1-x)^3]}{x^3} = \underline{\underline{\frac{4(1-x)^3}{x^3}}}$$

59. a) $\frac{\frac{2}{3} \cdot 15}{\frac{4}{5} \cdot 15} = \frac{10}{12} = \underline{\underline{\frac{5}{6}}}$

b) $\frac{\frac{2}{a} \cdot a}{b \cdot a} = \frac{2}{\underline{\underline{ab}}}$

c) $\frac{\frac{x \cdot y}{3} \cdot y}{\frac{3}{y} \cdot y} = \frac{xy}{\underline{\underline{3}}}$

d) $\frac{\frac{a}{a-1} \cdot \frac{a-1}{a-1}}{a+1} = \frac{a}{\underline{\underline{a^2-1}}}$

e) $\frac{\frac{2n}{3} \cdot \frac{n-2}{n-2}}{n-2} = \frac{2n^2-4n}{3} = \underline{\underline{\frac{2n(n-2)}{3}}}$

$$f) \frac{\frac{a}{x} \cdot x^3}{\frac{b}{x^3} \cdot x^3} = \frac{ax^2}{\underline{\underline{b}}}$$

$$60 \quad a) \frac{\left(\frac{3}{4} - \frac{1}{3}\right) \cdot 12}{\left(\frac{1}{2} + \frac{1}{3} - \frac{1}{4}\right) \cdot 12} = \frac{9-4}{6+4-3} = \frac{5}{\underline{\underline{7}}}$$

$$b) \frac{\left(1 + \frac{1}{a}\right) \cdot a}{\left(a + \frac{1}{a}\right) \cdot a} = \frac{a+1}{\underline{\underline{a^2+1}}}$$

$$c) \frac{\left(\frac{1}{m} - \frac{1}{n}\right) \cdot mn}{(m-n) \cdot mn} = \frac{(n-m)(-1)}{(m-n)mn(-1)} = \frac{\cancel{(n-m)}^1(-1)}{\cancel{(-m+n)}_1 mn} = -\frac{1}{\underline{\underline{mn}}}$$

$$e) \frac{\left(\frac{ac}{bd}\right) \cdot bd}{\left(\frac{a}{b} - \frac{a+b}{d}\right) \cdot bd} = \frac{ac}{ad - (a+b)b} = \frac{ac}{\underline{\underline{ad - ab - b^2}}}$$

$$f) \frac{\left(1 - \frac{1}{x}\right) \cdot x^3}{\left(\frac{1}{x^2} + \frac{1}{x^3}\right) \cdot x^3} = \frac{x^3 - x^2}{\underline{\underline{x+1}}}$$

$$61. \quad d) \frac{\left(\frac{a}{b} + 1\right) \cdot ab}{\left(\frac{b}{a} + 1\right) \cdot ab} = \frac{a^2 + ab}{b^2 + ab} = \frac{a \cancel{(a+b)}^1}{b \cancel{(b+a)}_1} = \frac{a}{\underline{\underline{b}}}$$

$$\frac{\left(\frac{4}{a^2} - \frac{4}{ab} + \frac{1}{b^2}\right) \cdot 2a^2b^2}{\left(\frac{1}{a} - \frac{1}{2b}\right) \cdot 2a^2b^2} = \frac{8b^2 - 8ab + 2a^2}{2ab^2 - a^2b} =$$

e) $\frac{2 \overbrace{(4b^2 - 4ab + a^2)}^{\text{Binom}}}{ab(2b-a)} = \frac{2(2b-a)^2}{ab \cancel{(2b-a)}} = \frac{2(2b-a)}{\underline{\underline{ab}}}$

$$\text{f) } \frac{\left(\frac{1}{a-1} + 1\right) \cdot (a-1)}{\left(\frac{a}{a-1} - 1\right) \cdot (a-1)} = \frac{1+a-1}{a-(a-1)} = \frac{a}{1} = \underline{\underline{a}}$$

$$62 \quad \text{a) } \frac{\left(u + \frac{1}{u-1}\right) \cdot (u-1)}{\left(u - \frac{1}{u-1}\right) \cdot (u-1)} = \frac{u^2 - u + 1}{\underline{\underline{u^2 - u - 1}}}$$

$$\text{b) } \frac{\left(\frac{a}{1-a} + 1\right) \cdot (1+a)(1-a)}{\left(\frac{1}{1+a} - 1\right) \cdot (1+a)(1-a)} = \frac{a+a^2+1-a^2}{1-a-(1-a^2)} = \frac{a+1}{1-a-1+a^2} = \frac{a+1}{a^2-a} = \frac{a+1}{\underline{\underline{a(a-1)}}}$$

$$\text{c) } \frac{\left(x-y - \frac{x-y}{x+y}\right) \cdot (y)(x+y)}{\left(\frac{x}{y} - \frac{x}{x+y}\right) \cdot (y)(x+y)} = \frac{(x-y)(y)(x+y) - (x-y)(y)}{x(x+y) - xy} =$$

$$\frac{(x-y)(y)(x+y-1)}{x(x+y-y)} = \frac{y(x-y)(x+y-1)}{\underline{\underline{x^2}}}$$

$$\text{d) } \frac{\left[\frac{b}{(b-1)(b+1)}\right] \cdot (b-1)(b+1)}{\left(\frac{1}{b+1} - \frac{1}{b-1}\right) \cdot (b-1)(b+1)} = \frac{b}{b-1-(b+1)} = \frac{b}{b-1-b-1} = \frac{b}{-2} = \underline{\underline{-\frac{b}{2}}}$$

$$\frac{\frac{a}{a+b} + \frac{b}{a-b}}{\frac{1}{a+b} - \frac{1}{b-a} \cdot \frac{(-1)}{(-1)}} = \frac{\left(\frac{a}{a+b} + \frac{b}{a-b}\right) \cdot (a+b)(a-b)}{\left(\frac{1}{a+b} - \frac{-1}{a-b}\right) \cdot (a+b)(a-b)} = \frac{a(a-b) + b(a+b)}{a-b - (-a-b)} =$$

$$\frac{a^2 - ab + ab + b^2}{a-b+a+b} = \frac{a^2 + b^2}{\underline{\underline{2a}}}$$

$$\frac{\left[\frac{(x+1)^2}{(x-1)(x+1)}\right] \cdot (x+1)(x-1)(x-1)}{\left[\frac{1}{x+1} - \frac{(1-x)(1+x)}{(x-1)(x-1)}\right] \cdot (x+1)(x-1)(x-1)} = \frac{(x+1)^2(x-1)}{(x-1)(x-1) - (1-x)(1+x)(x+1)} =$$

$$\text{f) } \frac{(x+1)^2(x-1)}{(x-1)(x-1) - [-(x-1)](1+x)(x+1)} = \frac{(x+1)^2 \cancel{(x-1)}}{\cancel{(x-1)}[x-1-[-1]](1+x)(x+1)} =$$

$$\frac{(x+1)^2}{x-1-(-1)(x^2+2x+1)} = \frac{(x+1)^2}{x-1-(-x^2-2x-1)} = \frac{(x+1)^2}{x-1+x^2+2x+1} = \frac{(x+1)^2}{\underline{\underline{x(x+3)}}$$

$$\text{63. c) } \frac{u - \frac{1}{u}}{u - \frac{u \cdot (u)}{u + \frac{1}{u}} \cdot (u)} = \frac{\left(u - \frac{1}{u}\right) \cdot (u^2 + 1) \cdot (u)}{\left[u - \frac{u^2}{(u^2 + 1)}\right] \cdot (u^2 + 1) \cdot (u)} =$$

$$\frac{u^2(u^2 + 1) - (u^2 + 1)}{u^2(u^2 + 1) - u^3} = \frac{(u^2 + 1)(u^2 - 1)}{u^2[(u^2 + 1) - u]} = \frac{u^4 - 1}{\underline{\underline{u^2(u^2 - u + 1)}}$$

$$\text{d) } \frac{a}{a + \frac{a \cdot (a-x)}{1 - \frac{a}{a-x}} \cdot (a-x)} = \frac{a}{a + \frac{a^2 - ax}{(a-x-a)}} = \frac{a \cdot (-x)}{\left(a + \frac{a^2 - ax}{-x}\right) \cdot (-x)} =$$

$$\frac{-ax}{-ax + a^2 - ax} = \frac{-ax}{a^2 - 2ax} = \frac{-\cancel{a}x}{\cancel{a}(a-2x)} = \frac{-x}{\underline{\underline{a-2x}}} = \frac{x}{\underline{\underline{2x-a}}}$$

64. a)
$$\frac{\frac{e^2 - 25}{e}}{\frac{-(e^2 - 2e - 15)}{(e-3)(e+3)} - 2e} = \frac{\frac{(e-5)(e+5)}{e}}{\frac{(e-5)\cancel{(e+3)}\cancel{(e-3)}}{\cancel{(e-3)}\cancel{(e+3)}} - 2e} =$$

$$\frac{(e-5)(e+5)}{e-5-2e} = \frac{(e-5)(e+5)(-1)}{e(-e-5)(-1)} = \frac{(-e+5)(e+5)}{e(e+5)} = \frac{5-e}{e}$$

b)
$$\frac{\frac{1}{a^2-1}}{a} = \frac{a}{(a-1)(a+1)} = \frac{a}{(a-1)(a+1)}$$

$$1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{a}}}$$

$$1 - \frac{1}{1 - \frac{a}{a-1}}$$

$$1 - \frac{a-1}{a-1-a}$$

$$\frac{\frac{a}{(a-1)(a+1)}}{1 - \frac{(a-1)(-1)}{(-1)(-1)}} = \frac{a}{(a-1)(a+1)} = \frac{1}{(a-1)(a+1)} = \frac{1}{a^2-1}$$

c)
$$\frac{\frac{2b-a}{ab}}{2a + \frac{(2b-3a)^3}{\cancel{9a^2+4b^2}} \cancel{(3a+2b)} - 12ab} = \frac{\frac{2b-a}{ab}}{2a + \frac{(2b-3a)^3}{\underbrace{9a^2-12ab+4b^2}_{\text{Reihenfolge ändern (einfacher): } 4b^2-12ab+9a^2}} - 12ab} = \frac{\frac{2b-a}{ab}}{2a + \frac{(2b-3a)^3}{(2b-3a)^2}} =$$

$$\frac{2b-a}{ab} = \frac{\cancel{2b-a}}{ab(\cancel{2b-a})} = \frac{1}{ab}$$