5 Rationale Zahlen

5.17 Übungen Frommenwiler

65. a)

$$\left(9x^3-6x^2-8x\right):(3x-4)=\underline{\underline{3x^2+2x}}$$
$$\underline{\overset{-}{+}9x^3\overset{+}{-}12x^2}$$
$$+6x^2-8x$$
$$\underline{\overset{-}{+}6x^2\overset{+}{-}8x}$$
$$0$$

b)

$$\left(6x^3+7x^2-36\right):(-2x+3)=\underline{\underline{-3x^2-8x-12}}$$
$$\underline{\overset{-}{+}6x^3\overset{+}{-}9x^2}$$
$$+16x^2-36$$
$$\underline{\overset{-}{+}16x^2\overset{+}{-}24x}$$
$$+24x-36$$
$$\underline{\overset{-}{+}24x\overset{+}{-}36}$$
$$0$$

c)

$$\left(26a^3+135a^2-30a+77\right):\left(13a^2-4a+7\right)=\underline{\underline{2a+11}}$$
$$\underline{\overset{-}{+}26a^3\overset{+}{-}8a^2\overset{-}{+}14a}$$
$$+143a^2-44a+77$$
$$\underline{\overset{-}{+}143a^2\overset{+}{-}44a\overset{-}{+}77}$$
$$0$$

d)

$$\left(81a^4-256b^4\right):\left(3a+4b\right)=\underline{\underline{27a^3-36a^2b+48ab^2-64b^3}}$$

$$\underline{\overset{-}{+}81a^4\overset{-}{+}108a^3b\qquad\qquad}$$

$$-108a^3b-256b^4$$

$$\underline{\overset{+}{-}108a^3b\overset{+}{-}144a^2b^2}$$

$$+144a^2b^2-256b^4$$

$$\underline{\overset{-}{+}144a^2b^2\overset{-}{+}192ab^3}$$

$$-192ab^3-256b^4$$

$$\underline{\overset{+}{-}192ab^3\overset{+}{-}256b^4}$$

$$0$$

Polynomdivision ungeordnet ausgeführt:

$$\left(8x^2-3y^2-3z^2+10xy+2xz+10yz\right):(2x+3y-z)=\underline{\underline{4x-y+3z}}$$

66. a)

$$\underline{\overset{-}{+}8x^2 \qquad \overset{-}{+}12xy\overset{+}{-}4xz}$$

$$-3y^2-3z^2-2xy+6xz+10yz$$

$$\underline{\overset{+}{-}3y^2 \qquad \overset{+}{-}2xy \qquad \overset{-}{+}yz}$$

$$-3z^2 \qquad +6xz+9yz$$

$$\underline{\overset{+}{-}3z^2 \qquad \overset{-}{+}6xz\overset{-}{+}9yz}$$

$$0$$

Polynomdivision geordnet ausgeführt:

$$\left(8x^2+10xy+2xz-3y^2+10yz-3z^2\right):(2x+3y-z)=\underline{\underline{4x-y+3z}}$$

a)

$$\underline{\overset{-}{+}8x^2\overset{-}{+}12xy\overset{+}{-}4xz}$$

$$-2xy+6xz-3y^2+10yz-3z^2$$

$$\underline{\overset{+}{-}2xy \qquad \overset{+}{-}3y^2\overset{-}{+}yz}$$

$$+6xz \qquad +9yz-3z^2$$

$$\underline{\overset{-}{+}6xz \qquad \overset{-}{+}9yz\overset{+}{-}3z^2}$$

$$0$$

b)

$$\left(4a^3-a^2-6a^2b+2ab-5ab^2+2b^3\right):(a-2b)=\underline{\underline{4a^2-a+2ab-b^2}}$$

$$\underline{\overset{-}{+}4a^3 \qquad \overset{+}{-}8a^2b}$$

$$-a^2+2a^2b+2ab-5ab^2+2b^3$$

$$\underline{\overset{+}{-}a^2 \qquad \overset{-}{+}2ab}$$

$$+2a^2b \qquad -5ab^2+2b^3$$

$$\underline{\overset{-}{+}2a^2b \qquad \overset{+}{-}4ab^2}$$

$$-ab^2+2b^3$$

$$\underline{\overset{+}{-}ab^2\overset{-}{+}2b^3}$$

$$0$$

67. a)

$$\left(3m^4 + m^3 - 2m^2 + 6\right) : \left(m^2 + 2m\right) = \underline{\underline{3m^2 - 5m + 8 + \frac{6 - 16m}{m^2 + 2m}}}$$

$$\begin{array}{l}
\underline{\overset{-}{+}3m^4 \overset{-}{+}6m^3} \\
\quad -5m^3 - 2m^2 + 6 \\
\quad \underline{\overset{+}{-}5m^3 \overset{+}{-}10m^2} \\
\qquad +8m^2 + 6 \\
\qquad \underline{\overset{-}{+}8m^2 \overset{-}{+}16m} \\
\qquad\quad -16m + 6
\end{array}$$

Polynomdivision ungeordnet ausgeführt:

$$\left(x^3 + y^3 + x^2y + 2xy^2\right) : (x + y) = \underline{\underline{x^2 + 2y^2 - \frac{y^3}{x + y}}}$$

c)

$$\begin{array}{l}
\underline{\overset{-}{+}x^3 \quad \overset{-}{+}x^2y} \\
\quad +y^3 \quad +2xy^2 \\
\quad \underline{\overset{-}{+}2y^3 \quad \overset{-}{+}2xy^2} \\
\quad -y^3
\end{array}$$

Polynomdivision geordnet ausgeführt:

$$\left(x^3 + x^2y + 2xy^2 + y^3\right) : (x + y) = \underline{\underline{x^2 + 2y^2 - \frac{y^3}{x + y}}}$$

c)

$$\begin{array}{l}
\underline{\overset{-}{+}x^3 \overset{-}{+}x^2y} \\
\qquad +2xy^2 + y^3 \\
\qquad \underline{\overset{-}{+}2xy^2 \overset{-}{+}2y^3} \\
\qquad\qquad -y^3
\end{array}$$

68. c)

$$\left(8n^3 - 2n^2 + nx + 1\right) : (2n + 1) = \underline{\underline{4n^2 - 3n + 1}}$$

$$\begin{array}{l}
\underline{\overset{-}{+}8n^3 \overset{-}{+}4n^2} \\
\quad -6n^2 + nx + 1 \\
\quad \underline{\overset{+}{-}6n^2 \overset{+}{-}3n} \\
\qquad +3n + nx + 1 \\
\qquad \underline{\overset{-}{+}2n \qquad \overset{-}{+}1} \\
\qquad +n + nx \qquad (\textbf{Rest})
\end{array}$$

Ansatz: **Rest = 0**

$n + nx = 0$

$nx = -n \quad \rightarrow \quad x = \frac{-n}{n} = \underline{\underline{-1}}$

Man muss -1 für x einsetzen, damit die Division aufgeht!

d)

$$(a^4 - a^3 + a^2 - a + x) : (a - 2) = \underline{\underline{a^3 + a^2 + 3a + 5}}$$

$$\underline{\overset{-}{+}a^4 \overset{+}{-} 2a^3}$$

$$+a^3 + a^2 - a + x$$

$$\underline{\overset{-}{+}a^3 \overset{+}{-} 2a^2}$$

$$+3a^2 - a + x$$

$$\underline{\overset{-}{+}3a^2 \overset{+}{-} 6a}$$

$$+5a + x$$

$$\underline{\overset{-}{+}5a \overset{+}{-} 10}$$

$$+x + 10 \qquad (\text{Rest})$$

Ansatz: **Rest = 0**

$x + 10 = 0$

$x = \underline{\underline{-10}}$

Man muss -10 für x einsetzen, damit die Division aufgeht!