6 Potenzieren

6.8 Übungen Frommenwiler

79. a) $(-2)^3 = \underline{\underline{-8}}$; $-2^3 = \underline{\underline{-8}}$; $-(-2)^3 = -(-8) = \underline{\underline{8}}$

b) $(-2)^4 = \underline{\underline{16}}$; $-2^4 = \underline{\underline{-16}}$; $-(-2)^4 = -(16) = \underline{\underline{-16}}$

c) $2^{-2} = \frac{1}{2^2} = \underline{\underline{\frac{1}{4}}}$; $(-2)^{-2} = \frac{1}{(-2)^2} = \underline{\underline{\frac{1}{4}}}$; $-2^{-2} = \frac{1}{-2^2} = \frac{1}{-4} = \underline{\underline{-\frac{1}{4}}}$

d) $2^{-3} = \frac{1}{2^3} = \underline{\underline{\frac{1}{8}}}$; $(-2)^{-3} = \frac{1}{(-2)^3} = \frac{1}{-8} = \underline{\underline{-\frac{1}{8}}}$; $-2^{-3} = \frac{1}{-2^3} = \frac{1}{-8} = \underline{\underline{-\frac{1}{8}}}$

e) $3^0 - 3^{-1} - 3^{-2} = 1 - \frac{1}{3} - \frac{1}{9} = \frac{9-3-1}{9} = \underline{\underline{\frac{5}{9}}}$

81. $x = -2 \rightarrow x^{-x} = (-2)^{-(-2)} = (-2)^2 = \underline{\underline{4}}$

$x = -1 \rightarrow x^{-x} = (-1)^{-(-1)} = (-1)^1 = \underline{\underline{-1}}$

$x = 1 \rightarrow x^{-x} = 1^{-1} = \frac{1}{1} = \underline{\underline{1}}$

$x = 2 \rightarrow x^{-x} = 2^{-2} = \frac{1}{2^2} = \underline{\underline{\frac{1}{4}}}$

$x = 3 \rightarrow x^{-x} = 3^{-3} = \frac{1}{3^3} = \underline{\underline{\frac{1}{27}}}$

82. a) $0.25^{-2} = \left(\frac{1}{4}\right)^{-2} = \frac{1^{-2}}{4^{-2}} = \frac{\frac{1}{1^2}}{\frac{1}{4^2}} = \frac{4^2}{1^2} = \left(\frac{4}{1}\right)^2 = \underline{\underline{16}}$

b) $0.2^{-3} = \left(\frac{1}{5}\right)^{-3} = \left(\frac{5}{1}\right)^{3} = \underline{\underline{125}}$

c) $-0.3^{-4} = -\left(\frac{3}{10}\right)^{-4} = -\left(\frac{10}{3}\right)^{4} = -\frac{10^4}{3^4} = -\underline{\underline{\frac{10'000}{81}}}$

d) $(-0.125)^{-3} = \left(-\frac{1}{8}\right)^{-3} = \left(-\frac{8}{1}\right)^{3} = \underline{\underline{-512}}$

e) $-0.75^{-3} = -\left(\frac{3}{4}\right)^{-3} = -\left(\frac{4}{3}\right)^{3} = \underline{\underline{-\frac{64}{27}}}$

f) $(1.5)^{-4} = \left(\frac{3}{2}\right)^{-4} = \left(\frac{2}{3}\right)^{4} = \underline{\underline{\frac{16}{81}}}$

g) $\left(2^{-1} + 3^{-1}\right)^{-1} = \left(\frac{1}{2} + \frac{1}{3}\right)^{-1} = \left(\frac{3+2}{6}\right)^{-1} = \underline{\underline{\frac{6}{5}}}$

h) $\left(2^0 - 2^{-2} - 2^{-3}\right)^{-2} = \left(1 - \frac{1}{4} - \frac{1}{8}\right)^{-2} = \left(\frac{8-2-1}{8}\right)^{-2} = \left(\frac{8}{5}\right)^{2} = \underline{\underline{\frac{64}{25}}}$

83. a) $\frac{1}{8} = \frac{1}{2^3} = \underline{\underline{2^{-3}}}$

b) $0.0001 = \frac{1}{10'000} = \frac{1}{10^4} = \underline{\underline{10^{-4}}}$

c) $\frac{1}{81} = \frac{1}{3^4} = \underline{\underline{3^{-4}}}$

d) $0.25 = \frac{1}{4} = \frac{1}{2^2} = \underline{\underline{2^{-2}}}$

e) $\frac{1}{625} = \frac{1}{5^4} = \underline{\underline{5^{-4}}}$

f) $0.0625 = \frac{1}{16} = \frac{1}{2^4} = \underline{\underline{2^{-4}}}$

g) $\frac{1}{216} = \frac{1}{6^3} = \underline{\underline{6^{-3}}}$

h) $0.008 = \frac{8}{1'000} = \frac{1}{125} = \frac{1}{5^3} = \underline{\underline{5^{-3}}}$

84. a) $3a^{-2} = \underline{\underline{\frac{3}{a^2}}}$

b) $(3a)^{-2} = \frac{1}{(3a)^2} = \underline{\underline{\frac{1}{9a^2}}}$

c) $-5m^{-5} = \underline{\underline{-\frac{5}{m^5}}}$

d) $(-4x)^{-2} = \frac{1}{(-4x)^2} = \underline{\underline{\frac{1}{16x^2}}}$

e) $ab^{-6} = \underline{\underline{\frac{a}{b^6}}}$

f) $-pq^{-4} = \underline{\underline{-\frac{p}{q^4}}}$

g) $2(a+b)^{-6} = \underline{\underline{\frac{2}{(a+b)^6}}}$

h) $2ab^{-5} = \underline{\underline{\frac{2a}{b^5}}}$

i) $3(ab)^{-2} = \underline{\underline{\frac{3}{(ab)^2}}} = \underline{\underline{\frac{3}{a^2b^2}}}$

85. a) $1 + x^{-2} = 1 + \underline{\underline{\frac{1}{x^2}}} = \underline{\underline{\frac{x^2+1}{x^2}}}$ **oder** $\frac{1+x^{-2}}{1} \cdot \frac{x^2}{x^2} = \underline{\underline{\frac{x^2+1}{x^2}}}$

b) $2a^{-1} - 3b^{-2} = \underline{\underline{\frac{2}{a} - \frac{3}{b^2}}} = \underline{\underline{\frac{2b^2 - 3a}{ab^2}}}$ **oder** $\frac{2a^{-1} - 3b^{-2}}{1} \cdot \frac{ab^2}{ab^2} = \underline{\underline{\frac{2b^2 - 3a}{ab^2}}}$

c) $\frac{1}{x^{-4}} = \frac{1}{\frac{1}{x^4}} = \underline{\underline{x^4}}$ **oder** $\frac{1}{x^{-4}} \cdot \frac{x^4}{x^4} = \underline{\underline{x^4}}$

d) $\frac{1}{3a^{-3}} = \underline{\underline{\frac{a^3}{3}}}$

e) $\frac{a}{2(a+b)^{-2}} = \underline{\underline{\frac{a(a+b)^2}{2}}}$

f) $\frac{x^{-2}}{y^{-3}} = \frac{\frac{1}{x^2}}{\frac{1}{y^3}} = \underline{\underline{\frac{y^3}{x^2}}}$

g) $\frac{5a^{-2}}{3b^{-1}} = \underline{\underline{\frac{5b}{3a^2}}}$

h) $\frac{ax^{-1}}{b^{-1}c^{-2}} = \underline{\underline{\frac{abc^2}{x}}}$

i) $\dfrac{a^0+a^{-1}}{a^{-1}-a^{-2}} = \dfrac{1+\frac{1}{a}}{\frac{1}{a}-\frac{1}{a^2}} = \dfrac{\frac{a+1}{a}}{\frac{a-1}{a^2}} = \dfrac{a+1}{a} \cdot \dfrac{a^2}{a-1} = \underline{\underline{\dfrac{a(a+1)}{a-1}}} = \underline{\underline{\dfrac{a^2+a}{a-1}}}$ **oder** $\dfrac{a^0+a^{-1}}{a^{-1}-a^{-2}} \cdot \dfrac{a^2}{a^2} = \underline{\underline{\dfrac{a^2+a}{a-1}}}$

86. a) $\left(\dfrac{1}{a}\right)^{-3} = \dfrac{1^{-3}}{a^{-3}} = \dfrac{\frac{1}{1^3}}{\frac{1}{a^3}} = \dfrac{a^3}{1^3} = \underline{\underline{a^3}}$

b) $\left(-\dfrac{a}{3}\right)^{-4} = \underline{\underline{\left(\dfrac{3}{a}\right)^4}} = \underline{\underline{\dfrac{81}{a^4}}}$

c) $-\left(\dfrac{m}{n}\right)^{-2} = \underline{\underline{-\left(\dfrac{n}{m}\right)^2}} = \underline{\underline{-\dfrac{n^2}{m^2}}}$

d) $\left(\dfrac{-1}{n}\right)^{-5} = \left(-\dfrac{n}{1}\right)^5 = \underline{\underline{-n^5}}$

e) $\left(\dfrac{a+1}{a-1}\right)^{-10} = \underline{\underline{\left(\dfrac{a-1}{a+1}\right)^{10}}} = \underline{\underline{\dfrac{(a-1)^{10}}{(a+1)^{10}}}}$

f) $-\left(-\dfrac{2}{x}\right)^{-4} = \underline{\underline{-\left(\dfrac{x}{2}\right)^4}} = \underline{\underline{-\dfrac{x^4}{16}}}$

87. a) $\dfrac{1}{3x} = \underline{\underline{(3x)^{-1}}}$

b) $\dfrac{2}{x^2} = \underline{\underline{2x^{-2}}}$

c) $\dfrac{a}{b} = \underline{\underline{ab^{-1}}}$

d) $\frac{n}{1+n^2} = \underline{\underline{n\left(1+n^2\right)^{-1}}}$

e) $\left(\frac{a}{b}\right)^4 = \underline{\underline{\left(ab^{-1}\right)^4}} = \underline{\underline{a^4b^{-4}}}$

f) $\frac{2p^2}{3q^3} = 2p^2\left(3q^3\right)^{-1} = \underline{\underline{2\cdot 3^{-1}p^2q^{-3}}}$

88. a) $1.4\cdot 3^5 - 0.9\cdot 3^5 = (1.4-0.9)\cdot 3^5 = \underline{\underline{0.5\cdot 3^5}}$

b) $3.6\cdot 5^9 - 12\cdot 5^9 + 1.2\cdot 5^9 = (3.6-12+1.2)\cdot 5^9 = \underline{\underline{-7.2\cdot 5^9}}$

c) $7^6 - 1.9\cdot 7^5 = (7-1.9)\cdot 7^5 = \underline{\underline{5.1\cdot 7^5}}$

d) $0.35\cdot 10^7 - 4.1\cdot 10^6 + 5\cdot 10^5 =$
$\left(0.35\cdot 10^2 - 4.1\cdot 10^1 + 5\right)\cdot 10^5 =$
$(35-41+5)\cdot 10^5 = \underline{\underline{-1\cdot 10^5}} = \underline{\underline{-10^5}}$

e) $3.8\cdot 10^9 - 0.5\cdot 10^{10} - 23\cdot 10^8 =$
$\left(3.8\cdot 10^1 - 0.5\cdot 10^2 - 23\right)\cdot 10^8 =$
$(38-50-23)\cdot 10^8 = \underline{\underline{-35\cdot 10^8}}$

f) $4\cdot 2^3 - 9.3\cdot 2^2 + 0.15\cdot 2^4 =$
$\left(4\cdot 2^1 - 9.3 + 0.15\cdot 2^2\right)\cdot 2^2 =$
$(8-9.3+0.6)\cdot 2^2 = \underline{\underline{-0.7\cdot 2^2}}$

g) $25\cdot 7^8 - 3.5\cdot 7^9 - 0.9\cdot 7^{10} =$
$\left(25 - 3.5\cdot 7^1 - 0.9\cdot 7^2\right)\cdot 7^8 =$
$(25-24.5-44.1)\cdot 7^8 = \underline{\underline{-43.6\cdot 7^8}}$

h) $21.5 \cdot 12^{13} - 1'520 \cdot 12^{11} + 25'120 \cdot 12^{10} =$

$\left(21.5 \cdot 12^{3} - 1'520 \cdot 12^{1} + 25'120\right) \cdot 12^{10} =$

$(37'152 - 18'240 + 25'120) \cdot 12^{10} = \underline{\underline{44'032 \cdot 12^{10}}}$

89. a) $6 \cdot 2^{n} - 11 \cdot 2^{n-1} = \left(6 - 11 \cdot 2^{-1}\right) \cdot 2^{n} = \left(6 - \frac{11}{2}\right) \cdot 2^{n} = \underline{\underline{0.5 \cdot 2^{n}}}$

b) $6 \cdot 5^{k+1} - 14 \cdot 5^{k} - 80 \cdot 5^{k-1} = \left(6 \cdot 5 - 14 - 80 \cdot 5^{-1}\right) \cdot 5^{k} = (30 - 14 - 16) \cdot 5^{k} = 0 \cdot 5^{k} = \underline{\underline{0}}$

c) $12 \cdot 4^{t+5} - 0.7 \cdot 4^{t+7} - 0.05 \cdot 4^{t+9} = \left(12 - 0.7 \cdot 4^{2} - 0.05 \cdot 4^{4}\right) \cdot 4^{t+5} = \underline{\underline{-12 \cdot 4^{t+5}}}$

d) $-\frac{5}{4} \cdot 5^{20} - \frac{6}{25} \cdot 5^{21} + 5 \cdot 5^{19} =$

$\left(-\frac{5}{4} \cdot 5 - \frac{6}{25} \cdot 5^{2} + 5\right) \cdot 5^{19} =$

$\left(-\frac{25}{4} - 6 + 5\right) \cdot 5^{19} = \underline{\underline{-7.25 \cdot 5^{19}}}$

e) $-\frac{3}{2} \cdot 6^{14} - 18 \cdot 6^{12} + 0.3 \cdot 6^{15} =$

$\left(-\frac{3}{2} \cdot 6^{3} - 18 \cdot 6 + 0.3 \cdot 6^{4}\right) \cdot 6^{11} =$

$(-324 - 108 + 388.8) \cdot 6^{11} = \underline{\underline{-43.2 \cdot 6^{11}}}$

f) $3.7 \cdot 3^{n+2} - 0.5 \cdot 3^{n+3} - 253 \cdot 3^{n} =$

$\left(3.7 \cdot 3^{2} - 0.5 \cdot 3^{3} - 253\right) \cdot 3^{n} =$

$(33.3 - 13.5 - 253) \cdot 3^{n} = \underline{\underline{-233.2 \cdot 3^{n}}}$

90. a) $2^{4} \cdot 5^{4} = (2 \cdot 5)^{4} = (10)^{4} = \underline{\underline{10'000}}$

b) $5^{5} \cdot 5^{7} : 5^{10} = 5^{5+7-10} = 5^{2} = \underline{\underline{25}}$

c) $\left(2\sqrt{3}\right)^4 = 2^4\left(\sqrt{3}\right)^4 = 16\cdot 9 = \underline{\underline{144}}$

d) $\left(\frac{\sqrt{3}}{2}\right)^6 = \frac{\left(\sqrt{3}\right)^6}{2^6} = \underline{\underline{\frac{27}{64}}}$

e) $\left(\sqrt{3}+1\right)^4\left(\sqrt{3}-1\right)^4 = \left[\left(\sqrt{3}+1\right)\left(\sqrt{3}-1\right)\right]^4 = (3-1)^4 = 2^4 = \underline{\underline{16}}$

f) $\left(3^{-3}\cdot 3^5\right):\left(3^2\cdot 3^{-4}\right) = 3^{-3+5}:3^{2-4} = 3^2:3^{-2} = 3^{2-(-2)} = 3^4 = \underline{\underline{81}}$

g) $\frac{8^9}{0.8^9} = \left(\frac{8}{0.8}\right)^9 = 10^9 = \underline{\underline{1'000'000'000}}$

h) $27\cdot 3^{-6} = 3^3\cdot 3^{-6} = 3^{3-6} = 3^{-3} = \underline{\underline{\frac{1}{27}}}$

i) $\left(\frac{3}{4}\right)^{-4}:\left(\frac{9}{8}\right)^{-4} = \left(\frac{\frac{3}{4}}{\frac{9}{8}}\right)^{-4} = \left(\frac{\cancel{3}}{\cancel{4}}\cdot\frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{9}}}\right)^{-4} = \left(\frac{3}{2}\right)^4 = \underline{\underline{\frac{81}{16}}}$

j) $\frac{2^3\cdot 3^5}{(2\cdot 3)^4} = 2^{3-4}\cdot 3^{5-4} = 2^{-1}\cdot 3 = \underline{\underline{\frac{3}{2}}}$

k) $\frac{2^3\cdot 15^3}{6^3} = \left(\frac{2\cdot 15}{6}\right)^3 = 5^3 = \underline{\underline{125}}$

l) $\left(\sqrt{5}-\sqrt{2}\right)^5\left(\sqrt{5}+\sqrt{2}\right)^5 = \left[\left(\sqrt{5}-\sqrt{2}\right)\left(\sqrt{5}+\sqrt{2}\right)\right]^5 = (5-2)^5 = 3^5 = \underline{\underline{243}}$

m) $\left(3^3\right)^5:\left(3^2\right)^6 = 3^{15}:3^{12} = 3^{15-12} = 3^3 = \underline{\underline{27}}$

n) $\frac{9^7}{27^3} = \frac{(3^2)^7}{(3^3)^3} = \frac{3^{14}}{3^9} = 3^{14-9} = 3^5 = \underline{\underline{243}}$

o) $\left(\frac{\sqrt{2}}{3\sqrt{3}}\right)^{-4} = \left(\frac{3\sqrt{3}}{\sqrt{2}}\right)^4 = \frac{3^4 \cdot 9}{4} = \underline{\underline{\frac{729}{4}}}$

p) $(-8)^5 \cdot (2^{-4})^4 = (-2^3)^5 \cdot 2^{-16} = -2^{15} \cdot 2^{-16} = -2^{15-16} = -2^{-1} = \underline{\underline{-\frac{1}{2}}}$

q) $\frac{(-4)^2 \cdot 2^4}{8^2} = \frac{(-2^2)^2 \cdot 2^4}{(2^3)^2} = \frac{2^4 \cdot 2^4}{2^6} = 2^{4+4-6} = 2^2 = \underline{\underline{4}}$

r) $\frac{4^n \cdot 25^{n+1}}{10^{2n+1}} = \frac{(2^2)^n \cdot (5^2)^{n+1}}{(2 \cdot 5)^{2n+1}} = \frac{2^{2n} \cdot 5^{2n+2}}{2^{2n+1} \cdot 5^{2n+1}} = 2^{2n-2n-1} \cdot 5^{2n+2-2n-1} = 2^{-1} \cdot 5^1 = \underline{\underline{\frac{5}{2}}}$

oder $\frac{4^n \cdot 25^n \cdot 25}{10^{2n} \cdot 10} = \frac{(4 \cdot 25)^n \cdot 25}{(10^2)^n \cdot 10} = \underline{\underline{\frac{5}{2}}}$

91. a) $5 \cdot 5^4 \cdot 5^{-2} = 5^{1+4-2} = \underline{\underline{5^3}}$

b) $3^4 \cdot 4^4 : 5^4 = \left(\frac{3 \cdot 4}{5}\right)^4 = \underline{\underline{\left(\frac{12}{5}\right)^4}}$

c) $(-2)^3 \cdot (-3)^3 \cdot (-4)^0 = [(-2)(-3)]^3 \cdot 1 = \underline{\underline{6^3}}$

d) $(-8)^4 : (-8^{10}) = \frac{8^4}{-8^{10}} = -8^{4-10} = \underline{\underline{-8^{-6}}} = \underline{\underline{-\frac{1}{8^6}}}$

e) $(-49) \cdot 7^{10} : 7^4 = -7^2 \cdot 7^{10} : 7^4 = -7^{2+10-4} = \underline{\underline{-7^8}}$

f) $(-10)^{\overset{\text{gerade}}{2n}} : \left(-2^{2n}\right) = \dfrac{10^{2n}}{-1 \cdot 2^{2n}} = -\left(\dfrac{10}{2}\right)^{2n} = \underline{\underline{-5^{2n}}}$

g) $3^5 \cdot 5^{-5} = \dfrac{3^5}{5^5} = \underline{\underline{\left(\dfrac{3}{5}\right)^5}}$

h) $2^n \cdot 3^{-n} \cdot 5^n = \dfrac{2^n \cdot 5^n}{3^n} = \underline{\underline{\left(\dfrac{10}{3}\right)^n}}$

i) $\left(-3^2\right)\color{red}{(-3)^3}\color{black} : 3^{-2} = \dfrac{-3^2 \cdot \left(\color{red}{-3^3}\color{black}\right)}{3^{-2}} = \dfrac{3^5}{3^{-2}} = 3^{5-(-2)} = \underline{\underline{3^7}}$

j) $4^k \cdot \left(\dfrac{1}{2}\right)^k \cdot \left(\dfrac{1}{3}\right)^{-k} = \left(\dfrac{4 \cdot 1}{2}\right)^k \cdot \left(\dfrac{3}{1}\right)^k = 2^k \cdot 3^k = (2 \cdot 3)^k = \underline{\underline{6^k}}$

k) $\underbrace{2^{10} + 2^{10}}_{\text{Kettenaddition}} = 2 \cdot 2^{10} = 2^{1+10} = \underline{\underline{2^{11}}}$

l) $\underbrace{3^n + 3^n + 3^n}_{\text{Kettenaddition}} = 3 \cdot 3^n = 3^{1+n} = \underline{\underline{3^{n+1}}}$

92. a) $a^n \cdot a^n = a^{n+n} = \underline{\underline{a^{2n}}}$

b) $t^{3n} : t^n = t^{3n-n} = \underline{\underline{t^{2n}}}$

c) $\dfrac{(x+y)^5}{x^5} = \underline{\underline{\left(\dfrac{x+y}{x}\right)^5}}$

d) $2^n \cdot n^n = \underline{\underline{(2n)^n}}$

e) $a^k : a = \underline{\underline{a^{k-1}}}$

f) $a^0 \cdot a \cdot a^{2n} \cdot a^{3n} = a^{0+1+2n+3n} = \underline{\underline{a^{5n+1}}}$

g) $a^{n+1} : a^{n-1} = a^{n+1-(n-1)} = a^{n+1-n+1} = \underline{\underline{a^2}}$

h) $(x-1)^m \cdot x^m = \left[(x-1)x\right]^m = \underline{\underline{(x^2-x)^m}}$

i) $\left(\frac{1}{a}\right)^x : \left(\frac{1}{b}\right)^x = \left(\frac{\frac{1}{a}}{\frac{1}{b}}\right)^x = \left(\frac{1}{a} \cdot \frac{b}{1}\right)^x = \underline{\underline{\left(\frac{b}{a}\right)^x}}$

j) $a : a^7 = a^{1-7} = \underline{\underline{a^{-6}}} = \underline{\underline{\frac{1}{a^6}}}$

k) $a^{x+1} \cdot a^{x-1} = a^{x+1+x-1} = \underline{\underline{a^{2x}}}$

l) $(u^2-v^2)^m : (u-v)^m = \left(\frac{u^2-v^2}{u-v}\right)^m = \left[\frac{\cancel{(u-v)}(u+v)}{\cancel{u-v}}\right]^m = \underline{\underline{(u+v)^m}}$

93. a) $(a+b)^3 \cdot (a-b)^3 = \left[(a+b)\cdot(a-b)\right]^3 = \underline{\underline{(a^2-b^2)^3}}$

b) $a^{m+n} : a^{m-n} = a^{m+n-(m-n)} = a^{m+n-m+n} = \underline{\underline{a^{2n}}}$

c) $\left(\frac{a}{b}\right)^5 : \left(\frac{b}{2}\right)^5 = \left(\frac{\frac{a}{b}}{\frac{b}{2}}\right)^5 = \left(\frac{a}{b} \cdot \frac{2}{b}\right)^5 = \underline{\underline{\left(\frac{2a}{b^2}\right)^5}} = \frac{2^5 a^5}{b^{10}} = \underline{\underline{\frac{32a^5}{b^{10}}}}$

d) $\underbrace{(-a^5)}_{\text{negativ}} \cdot \underbrace{(-a^6)}_{\text{negativ}} = a^{5+6} = \underline{\underline{a^{11}}}$

e) $\underbrace{(-a)^4}_{\text{positiv}} \cdot \underbrace{(-b^4)}_{\text{negativ}} = \underline{\underline{-(ab)^4}} = \underline{\underline{-a^4b^4}}$

f) $(a-1)^m : (1-a)^m = \dfrac{(a-1)^m}{[(-1)(-1+a)]^m} = \left[\dfrac{\cancel{a-1}}{(-1)\,\cancel{(a-1)}}\right]^m = \left(\dfrac{1}{-1}\right)^m = \underline{\underline{(-1)^m}}$

g) $u \cdot u^{2k-3} : u^{1-k} = u^{1+2k-3-(1-k)} = u^{1+2k-3-1+k} = \underline{\underline{u^{3k-3}}}$

h) $\underbrace{(-x)^3}_{\text{negativ}} \cdot \underbrace{(-x^4)}_{\text{negativ}} = x^{3+4} = \underline{\underline{x^7}}$

i) $\left(\dfrac{x}{y}\right)^7 : \underbrace{\left(-\dfrac{x}{y}\right)^4}_{\text{positiv}} = \left(\dfrac{x}{y}\right)^7 : \left(\dfrac{x}{y}\right)^4 = \left(\dfrac{x}{y}\right)^{7-4} = \underline{\underline{\left(\dfrac{x}{y}\right)^3}}$

oder

$\left(\dfrac{x}{y}\right)^7 : \left(-\dfrac{x}{y}\right)^4 = \dfrac{x^7}{y^7} : \dfrac{x^4}{y^4} = \dfrac{x^7}{y^7} \cdot \dfrac{y^4}{x^4} = x^{7-4} \cdot y^{4-7} = x^3 \cdot y^{-3} = \underline{\underline{\left(\dfrac{x}{y}\right)^3}}$

j) $(-a)^4 \cdot (-a^6) = a^4 \cdot (-a^6) = -a^{4+6} = \underline{\underline{-a^{10}}}$

k) $(5a)^n : (5a^n) = \dfrac{5^n a^n}{5a^n} = \underline{\underline{5^{n-1}}}$

l) $(ab-ac)^k : (bc-c^2)^k = \left[\dfrac{a\,\cancel{(b-c)}}{c\,\cancel{(b-c)}}\right]^k = \underline{\underline{\left(\dfrac{a}{c}\right)^k}}$

94. a) $x^{2n+6} : (-x)^{2n-6} = \dfrac{x^{2n+6}}{[(-1)(x)]^{2n-6}} = \dfrac{x^{2n+6}}{\underbrace{(-1)^{2n-6}}_{\text{positiv, weil } n\in\mathbf{Z}}\,(x)^{2n-6}} = x^{2n+6-(2n-6)} = x^{2n+6-2n+6} = \underline{\underline{x^{12}}}$

b) $\left(\dfrac{1}{a}\right)^n : \left(\dfrac{a}{3}\right)^n = \left(\dfrac{1}{a} \cdot \dfrac{3}{a}\right)^n = \underline{\underline{\left(\dfrac{3}{a^2}\right)^n}} = \underline{\underline{\dfrac{3^n}{a^{2n}}}}$

c) $a^b \cdot b^a$ (kann nicht anders geschrieben werden)

d) $$\frac{a^{3n}}{(-a)^{2n+1}} = \frac{a^{3n}}{[(-1)\cdot(a)]^{2n+1}} = \frac{a^{3n}}{\underbrace{(-1)^{2n+1}}_{\text{negativ, weil } n\in Z}\ (a)^{2n+1}} = -a^{3n-(2n+1)} = -a^{3n-2n-1} = \underline{\underline{-a^{n-1}}}$$

e) $$\left(\frac{a}{b}\right)^n \cdot \left(\frac{b}{a}\right)^{2n-1} = \frac{a^n}{b^n}\cdot\frac{b^{2n-1}}{a^{2n-1}} = a^{n-(2n-1)}\cdot b^{2n-1-n} = a^{n-2n+1}\cdot b^{n-1} = \underline{\underline{a^{-n+1}\cdot b^{n-1}}} = \frac{b^{n-1}}{a^{n-1}} = \underline{\underline{\left(\frac{b}{a}\right)^{n-1}}}$$

f) $$(x-y)^5\cdot(y-x) = [(-1)(y-x)]^5\cdot(y-x) = -1\cdot(y-x)^{5+1} =$$
$$\underline{\underline{-(y-x)^6}} = -[(-1)(x-y)]^6 = \underline{\underline{-(x-y)^6}}$$

g) $$\underbrace{(-a)^{-6}}_{\text{positiv}}\cdot\underbrace{(-a^8)}_{\text{negativ}} = a^{-6}\cdot(-a^8) = -a^{-6+8} = \underline{\underline{-a^2}}$$

h) $$3^m\cdot(-3)^n = 3^m\cdot[(-1)(3)]^n = 3^m\cdot(-1)^n\cdot 3^n = \underbrace{\underline{\underline{\underbrace{(-1)^n}_{\text{positiv od. negativ}}\cdot 3^{m+n}}}}_{\text{keine Vereinfachung}}$$

i) $$\left(\frac{2}{3}\right)^n\cdot\left(\frac{3}{4}\right)^{-n} = \left(\frac{2}{3}\right)^n\cdot\left(\frac{4}{3}\right)^n = \left(\frac{2\cdot4}{3\cdot3}\right)^n = \underline{\underline{\left(\frac{8}{9}\right)^n}}$$

j) $$(a-b)^3(b-a)^6 = (a-b)^3[(-1)(a-b)]^6 = (a-b)^3(-1)^6(a-b)^6 = (a-b)^{3+6} = \underline{\underline{(a-b)^9}}$$

k) $$(u-v)^n:(v-u)^n = \frac{(u-v)^n}{[(-1)(u-v)]^n} = \left[\frac{\cancel{u-v}}{(-1)\,\cancel{(u-v)}}\right]^n = \left(\frac{1}{-1}\right)^n = \underline{\underline{(-1)^n}}$$

l) $$\underbrace{(-x)^{2n-1}}^{\text{ungerade, weil } n\in Z} : (-x^{3n+1}) = \frac{-x^{2n-1}}{-x^{3n+1}} = x^{2n-1-(3n+1)} = x^{2n-1-3n-1} = \underline{\underline{x^{-n-2}}}$$

95. a) $\left(-a^3\right)^4 = \underline{\underline{a^{12}}}$

b) $\left(-a^4\right)^3 = \underline{\underline{-a^{12}}}$

c) $\left[-\left(x^{-1}\right)^{-2}\right]^6 = \left[-x^2\right]^6 = \underline{\underline{x^{12}}}$

d) $\left(-u^{-2}\right)^{\overset{\text{ungerade}}{-3}} = \underline{\underline{-u^6}}$

e) $-\underbrace{\left(y^{-10}\right)^0}_{(\cdots)^0=1} = \underline{\underline{-1}}$

f) $\left[-(-b)^3\right]^{-4} = \left[-\left(-b^3\right)\right]^{-4} = \left[b^3\right]^{-4} = \underline{\underline{b^{-12}}} = \underline{\underline{\frac{1}{b^{12}}}}$

96. a) $\left(3a^2\right)^4 = 3^4 a^8 = \underline{\underline{81a^8}}$

b) $\left(\frac{2}{3}c^4\right)^{-2} = \left(\frac{3}{2}\right)^2 c^{-8} = \underline{\underline{\frac{9}{4c^8}}}$

c) $\left(\frac{1}{2}a^2 \cdot \frac{1}{3}b^3\right)^2 = \left(\frac{a^2b^3}{6}\right)^2 = \underline{\underline{\frac{a^4b^6}{36}}}$

d) $\left(\frac{a^{-1} \cdot b^{-3}}{c^2}\right)^{-2} = \frac{a^2 \cdot b^6}{c^{-4}} = \underline{\underline{a^2 \cdot b^6 \cdot c^4}}$

e) $\left(5a^n \cdot b^{n+1}\right)^n = \underline{\underline{5^n a^{n^2} b^{n^2+n}}}$

f) $\left[-\left(2a^{-1}\right)^{-2}\right]^3 = \left[-\left(2^{-2}a^2\right)\right]^3 = -2^{-6}a^6 = -\frac{a^6}{2^6} = \underline{\underline{-\frac{a^6}{64}}}$

g) $\left(2a^2b\right)^3\left(3ab^2\right)^3 = \left(2a^2b\cdot 3ab^2\right)^3 = 6^3a^9b^9 = \underline{\underline{216a^9b^9}}$

h) $\left(\frac{2a^{-1}b^2}{3ac^{-2}}\right)^{-3} = \frac{2^{-3}a^3b^{-6}}{3^{-3}a^{-3}c^6} = \frac{3^3a^6}{2^3b^6c^6} = \underline{\underline{\frac{27a^6}{8b^6c^6}}}$

97. a) $\left(\frac{3a^{-2}b^2}{4a^{-4}b^3}\right)^{-2} : \left(\frac{2a^{-1}}{3ab^{-2}}\right)^3 = \frac{3^{-2}a^4b^{-4}}{4^{-2}a^8b^{-6}} : \frac{2^3a^{-3}}{3^3a^3b^{-6}} = \frac{3^{-2}a^4b^{-4}}{\underbrace{4^{-2}}_{2^{-4}}a^8b^{-6}}\cdot\frac{3^3a^3b^{-6}}{2^3a^{-3}} =$

$\frac{3^{-2+3}a^{4+3-8+3}b^{-4-6+6}}{2^{-4+3}} = \frac{3a^2b^{-4}}{2^{-1}} = \underline{\underline{6a^2b^{-4}}}$

b) $\left(\frac{u}{v}\right)^n\cdot\left(\frac{v}{u}\right)^{3n+4} : \left(\frac{-v}{u}\right)^{2n+1} = \frac{u^n\cdot v^{3n+4}\cdot u^{2n+1}}{v^n\cdot u^{3n+4}\cdot(-v)^{2n+1}} =$

$\frac{u^n\cdot v^{3n+4}\cdot u^{2n+1}}{v^n\cdot u^{3n+4}\cdot\left[(-1)(v)\right]^{2n+1}} = \frac{u^{n+2n+1-3n-4}\cdot v^{3n+4-n-2n-1}}{\underbrace{(-1)^{2n+1}}_{\text{negativ, weil } n\in Z}} = -u^{-3}\cdot v^3 = \underline{\underline{-\left(\frac{v}{u}\right)^3}}$

c) $\left(\frac{x-1}{x+1}\right)^n\left(\frac{x+1}{1-x}\right)^{3n} = \left(\frac{x-1}{x+1}\right)^n\left[\frac{x+1}{-(x-1)}\right]^{3n} = \left(\frac{x-1}{x+1}\right)^n \underbrace{\left[\frac{-(x+1)}{(x-1)}\right]^{3n}}_{\text{zur Erinnerung: } -\frac{a}{b}=\frac{-a}{b}=\frac{a}{-b}} =$

$\frac{(x-1)^n}{(x+1)^n}\cdot\frac{(-1)^{3n}(x+1)^{3n}}{(x-1)^{3n}} = (x-1)^{n-3n}\cdot(x+1)^{3n-n}\cdot\underbrace{(-1)^{3n}}_{\left[(-1)^3\right]^n} =$

$(x-1)^{-2n}\cdot(x+1)^{2n}\cdot\underbrace{(-1)^n}_{\text{positiv oder negativ}} = \underline{\underline{(-1)^n\left(\frac{x+1}{x-1}\right)^{2n}}}$

oder (je nach Vorgehensweise)

$\left(\frac{x-1}{x+1}\right)^n\left(\frac{x+1}{1-x}\right)^{3n} = \left[\frac{-(1-x)}{x+1}\right]^n\left(\frac{x+1}{1-x}\right)^{3n} = \frac{(-1)^n(1-x)^n}{(x+1)^n}\cdot\frac{(x+1)^{3n}}{(1-x)^{3n}} =$

$(-1)^n(1-x)^{-2n}(x+1)^{2n} = (-1)^n\frac{(x+1)^{2n}}{\underbrace{(1-x)^{2n}}_{\underbrace{(-1)^{2n}}_{\text{positiv}}(x-1)^{2n}}} = \underline{\underline{\underbrace{(-1)^n}_{\text{positiv oder negativ}}\left(\frac{x+1}{x-1}\right)^{2n}}} = \underline{\underline{(-1)^n\left(\frac{x+1}{1-x}\right)^{2n}}}$

d) $$\frac{\left(\frac{a+b}{a}\right)^{2n}\cdot\left(\frac{b}{a+b}\right)^{n+1}\cdot\left[(a-b)(a+b)\right]^{n}}{a\left(\frac{a-b}{b}\right)^{n}}=\frac{\frac{(a+b)^{2n}}{a^{2n}}\cdot\frac{b^{n+1}}{(a+b)^{n+1}}\cdot(a-b)^{n}\cdot(a+b)^{n}}{a\cdot\frac{(a-b)^{n}}{b^{n}}}=$$

$$\frac{\frac{(a+b)^{2n}\cdot b^{n+1}\cdot(a-b)^{n}\cdot(a+b)^{n}}{a^{2n}\cdot(a+b)^{n+1}}}{\frac{a\cdot(a-b)^{n}}{b^{n}}}=\frac{(a+b)^{2n}\cdot b^{n+1}\cdot\cancel{(a-b)^{n}}\cdot(a+b)^{n}}{a^{2n}\cdot(a+b)^{n+1}}\cdot\frac{b^{n}}{a\cdot\cancel{(a-b)^{n}}}=$$

$$\frac{(a+b)^{2n+n-n-1}\cdot b^{n+1+n}}{a^{2n+1}}=\frac{(a+b)^{2n-1}\cdot b^{2n+1}}{a^{2n+1}}=\underline{\underline{(a+b)^{2n-1}\left(\frac{b}{a}\right)^{2n+1}}}$$

98. a) $$\frac{f^{-(2g+3)}}{(-f)^{-(4g+3)}}=\frac{f^{-2g-3}}{[-(f)]^{-4g-3}}=\frac{f^{-2g-3}}{\underbrace{(-1)^{-4g-3}}_{\text{negativ, weil } g\in Z}\cdot(f)^{-4g-3}}=-f^{-2g-3-(-4g-3)}=-f^{-2g-3+4g+3}=\underline{\underline{-f^{2g}}}$$

b) $$\frac{(5u-w)^{-7}}{(6u+2w)^{-7}}:\left[\frac{(6u+2w)^{-3}}{(5u-w)^{-3}}\cdot\frac{(5u-w)^{6}}{(6u+2w)^{6}}\right]=$$

$$\frac{(5u-w)^{-7}}{(6u+2w)^{-7}}\cdot\frac{(5u-w)^{-3}\cdot(6u+2w)^{6}}{(6u+2w)^{-3}\cdot(5u-w)^{6}}=$$

$$(5u-w)^{-7-3-6}\cdot(6u+2w)^{6+7+3}=(5u-w)^{-16}(6u+2w)^{16}=\underline{\underline{\left(\frac{6u+2w}{5u-w}\right)^{16}}}$$

c) $$\frac{5a^{6}(r+s)^{-2}}{6b^{5}c^{7}}\cdot\frac{3b^{3}c^{6}(x+y)^{3}(r+s)^{6}}{a^{4}}=$$

$$\frac{15a^{6-4}b^{3-5}c^{6-7}(r+s)^{-2+6}(x+y)^{3}}{6}=$$

$$\frac{\overset{5}{\cancel{15}}\,a^{2}b^{-2}c^{-1}(r+s)^{4}(x+y)^{3}}{\underset{2}{\cancel{6}}}=\underline{\underline{\frac{5a^{2}(r+s)^{4}(x+y)^{3}}{2b^{2}c}}}$$

103. a) $10^{2x}=10^{8}\ \rightarrow\ 2x=8\ \rightarrow\ x=\underline{\underline{4}}$

b) $a^{x}\cdot a^{x-2}=a^{x+1}\ \rightarrow\ a^{x+x-2}=a^{x+1}\ \rightarrow\ a^{2x-2}=a^{x+1}\ \rightarrow\ 2x-2=x+1\ \rightarrow\ x=\underline{\underline{3}}$

c) $2^x = \underbrace{16}_{2^4} \cdot 2^7 \rightarrow 2^x = 2^{4+7} \rightarrow 2^x = 2^{11} \rightarrow x = \underline{\underline{11}}$

d) $2^{(x^2)} = \underbrace{16}_{2^4} \rightarrow 2^{(x^2)} = 2^4 \rightarrow x^2 = 4 \rightarrow x_{1,2} = \underline{\underline{\pm 2}}$

e) $\left(100^{100}\right)^{100} = 10^x \rightarrow \underbrace{100}_{10^2}{}^{100\cdot 100} = 10^x \rightarrow 10^{2\cdot 100\cdot 100} = 10^x \rightarrow x = \underline{\underline{20'000}}$

f) $2^x = \frac{8^4}{4^8} \rightarrow 2^x = \frac{\left(2^3\right)^4}{\left(2^2\right)^8} \rightarrow 2^x = \frac{2^{12}}{2^{16}} = 2^{12-16} = 2^{-4} \rightarrow x = \underline{\underline{-4}}$

g) $\left(3^x\right)^6 = \frac{1}{81} \rightarrow 3^{6x} = \frac{3^0}{3^4} = 3^{0-4} = 3^{-4} \rightarrow 6x = -4 \rightarrow x = \underline{\underline{-\frac{2}{3}}}$

h) $\frac{25^k}{125^3} = 5^x \rightarrow \frac{\left(5^2\right)^k}{\left(5^3\right)^3} = 5^x \rightarrow \frac{5^{2k}}{5^9} = 5^{2k-9} = 5^x \rightarrow x = \underline{\underline{2k-9}}$

i) $13\cdot 3^n - 4\cdot 3^n = 3^x \rightarrow 3^n \underbrace{(13-4)}_{9} = 3^x \rightarrow 3^n \cdot 3^2 = 3^x \rightarrow 3^{n+2} = 3^x \rightarrow x = \underline{\underline{n+2}}$

j) $3\cdot 2^{10} - 10\cdot 2^8 = 2^x \rightarrow 2^8 \underbrace{\left(3\cdot 2^2 - 10\right)}_{12-10} = 2^x \rightarrow 2^8 \cdot 2 = 2^x \rightarrow 2^{8+1} = 2^x \rightarrow x = \underline{\underline{9}}$

k) $50\cdot 5^{n-1} + 3\cdot 5^{n+1} = 5^x \rightarrow 5^n \underbrace{\left(50\cdot 5^{-1} + 3\cdot 5\right)}_{\frac{50}{5}+15=25=5^2} = 5^x \rightarrow 5^n \cdot 5^2 = 5^x \rightarrow 5^{n+2} = 5^x \rightarrow x = \underline{\underline{n+2}}$

l) $\frac{2^{3x+1}\cdot 4^{5x-3}}{8^{4x-1}} = 32 \rightarrow \frac{2^{3x+1}\cdot\left(2^2\right)^{5x-3}}{\left(2^3\right)^{4x-1}} = 2^5 \rightarrow \frac{2^{3x+1}\cdot 2^{10x-6}}{2^{12x-3}} = 2^5$

$2^{3x+1+10x-6-12x+3} = 2^5 \rightarrow 2^{x-2} = 2^5 \rightarrow x = \underline{\underline{7}}$

m) $9^{(9^9)} : \left(9^9\right)^9 = 9^x \rightarrow \frac{9^{(9^9)}}{9^{(9^2)}} = 9^x \rightarrow 9^{(9^9)-(9^2)} = 9^x \rightarrow x = \underline{\underline{9^9 - 9^2}} = 387'420'408$

104. a) $\frac{1-x^2}{x^8}+\frac{1+x}{x^6}-\frac{1}{x^5}=\frac{1-x^2}{x^8}+\frac{1+x}{x^6}\cdot\frac{x^2}{x^2}-\frac{1}{x^5}\cdot\frac{x^3}{x^3}=\frac{1-x^2+x^2+x^3-x^3}{x^8}=\underline{\underline{\frac{1}{x^8}}}$

b) $\frac{1}{a^b}+\frac{1}{a^{b+1}}=\frac{1}{a^b}\cdot\frac{a^1}{a^1}+\frac{1}{a^{b+1}}=\underline{\underline{\frac{a+1}{a^{b+1}}}}$

c) $\frac{1}{b^x}-\frac{1}{b^y}=\frac{1}{b^x}\cdot\frac{b^y}{b^y}-\frac{1}{b^y}\cdot\frac{b^x}{b^x}=\underline{\underline{\frac{b^y-b^x}{b^{x+y}}}}$

d) $\frac{1}{a^n}-\frac{2}{a^{n-1}}+\frac{1}{a^{n-2}}=\frac{1}{a^n}\cdot\frac{a^{-2}}{a^{-2}}-\frac{2}{a^{n-1}}\cdot\frac{a^{-1}}{a^{-1}}+\frac{1}{a^{n-2}}=$

$\frac{a^{-2}-2a^{-1}+1}{a^n\cdot a^{-2}}=\frac{a^{-2}-2a^{-1}+1}{a^n}\cdot\frac{a^2}{1}=\frac{\overbrace{1-2a+a^2}^{\text{Binom}}}{a^n}=\underline{\underline{\frac{(a-1)^2}{a^n}}}$

oder (einfacher)

$\frac{1}{a^n}-\frac{2}{a^{n-1}}+\frac{1}{a^{n-2}}=\frac{1}{a^n}-\frac{2}{a^{n-1}}\cdot\frac{a^1}{a^1}+\frac{1}{a^{n-2}}\cdot\frac{a^2}{a^2}=\frac{\overbrace{1-2a+a^2}^{\text{Binom}}}{a^n}=\underline{\underline{\frac{(a-1)^2}{a^n}}}$

e) $\frac{x^5+1}{x^{m+2}}-\frac{2x^2-2}{x^m}+\frac{2-x}{x^{m-2}}=\frac{x^5+1}{x^{m+2}}\cdot\frac{x^{-2}}{x^{-2}}-\frac{2x^2-2}{x^m}+\frac{2-x}{x^{m-2}}\cdot\frac{x^2}{x^2}=$

$\frac{x^3+x^{-2}}{x^m}-\frac{2x^2-2}{x^m}+\frac{2x^2-x^3}{x^m}=\frac{x^3+x^{-2}-2x^2+2+2x^2-x^3}{x^m}=$

$\frac{\frac{1}{x^2}+2}{x^m}=\frac{\frac{1+2x^2}{x^2}}{x^m}=\frac{1+2x^2}{x^2}\cdot\frac{1}{x^m}=\underline{\underline{\frac{2x^2+1}{x^{m+2}}}}$

oder

$\frac{x^5+1}{x^{m+2}}-\frac{2x^2-2}{x^m}+\frac{2-x}{x^{m-2}}=\frac{x^5+1}{x^{m+2}}-\frac{2x^2-2}{x^m}\cdot\frac{x^2}{x^2}+\frac{2-x}{x^{m-2}}\cdot\frac{x^4}{x^4}=$

$\frac{x^5+1}{x^{m+2}}-\frac{2x^4-2x^2}{x^{m+2}}+\frac{2x^4-x^5}{x^{m+2}}=\frac{x^5+1-2x^4+2x^2+2x^4-x^5}{x^{m+2}}=\underline{\underline{\frac{2x^2+1}{x^{m+2}}}}$

f) $$\frac{1-2e^2}{e^p}+\frac{2-3e^2}{e^{p-2}}+\frac{3}{e^{p-4}}=\frac{1-2e^2}{e^p}+\frac{2-3e^2}{e^{p-2}}\cdot\frac{e^2}{e^2}+\frac{3}{e^{p-4}}\cdot\frac{e^4}{e^4}=$$

$$\frac{1-2e^2}{e^p}+\frac{2e^2-3e^4}{e^p}+\frac{3e^4}{e^p}=\frac{1-2e^2+2e^2-3e^4+3e^4}{e^p}=\underline{\underline{\frac{1}{e^p}}}$$

105. a) $$a^{n+2}-a^n=a^n\underbrace{\left(a^2-1\right)}_{\text{Binom}}=\underline{\underline{a^n\left(a+1\right)\left(a-1\right)}}$$

b) $$a^n-a^{n-2}=a^n\left(1-a^{-2}\right)=a^n\left(1-\frac{1}{a^2}\right)=a^n\left(\frac{a^2-1}{a^2}\right)=\underline{\underline{a^{n-2}\left(a+1\right)\left(a-1\right)}}$$

oder

$$a^n-a^{n-2}=\underbrace{a^{n-2}\left(a^{n-n+2}-1\right)}_{\text{weil}}=a^{n-2}\left(a^2-1\right)=\underline{\underline{a^{n-2}\left(a+1\right)\left(a-1\right)}}$$

$$a^{n-2}\left(\frac{a^n-a^{n-2}}{a^{n-2}}\right)=a^{n-2}\left(\frac{a^n}{a^{n-2}}-\frac{a^{n-2}}{a^{n-2}}\right)$$

c) $$x^{12}-x^8=x^8\underbrace{\left(x^4-1\right)}_{\text{Binom}}=x^8\left(x^2+1\right)\underbrace{\left(x^2-1\right)}_{\text{Binom}}=\underline{\underline{x^8\left(x^2+1\right)\left(x+1\right)\left(x-1\right)}}$$

d) $$m^3-2m^4+m^5=m^3\left(1-2m+m^2\right)=\underline{\underline{m^3\left(m-1\right)^2}}$$

e) $$p^{2n-1}-p^{2n+1}=p^{2n}\cdot p^{-1}-p^{2n}\cdot p=p^{2n}\left(\frac{1}{p}-p\right)=p^{2n}\left(\frac{1-p^2}{p}\right)=\underline{\underline{p^{2n-1}\left(1-p\right)\left(1+p\right)}}$$

oder

$$p^{2n-1}-p^{2n+1}=\underbrace{p^{2n-1}\left(1-p^{2n+1-2n+1}\right)}_{\text{weil}}=p^{2n-1}\left(1-p^2\right)=\underline{\underline{p^{2n-1}\left(1-p\right)\left(1+p\right)}}$$

$$p^{2n-1}\left(\frac{p^{2n-1}-p^{2n+1}}{p^{2n-1}}\right)=p^{2n-1}\left(\frac{p^{2n-1}}{p^{2n-1}}-\frac{p^{2n+1}}{p^{2n-1}}\right)$$

f) $$e^{x-1}+2e^x+e^{x+1}=e^{x-1}\left(1+2e^{x-x+1}+e^{x+1-x+1}\right)=e^{x-1}\left(1+2e+e^2\right)=\underline{\underline{e^{x-1}\left(e+1\right)^2}}$$

g) $$x^{2n}+2x^ny^n+y^{2n}=\left(x^n\right)^2+2x^ny^n+\left(y^n\right)^2=\underline{\underline{\left(x^n+y^n\right)^2}}$$

h) $$a^{2n}-1=\left(a^n\right)^2-1=\underline{\underline{\left(a^n+1\right)\left(a^n-1\right)}}$$

i) $x^{2n}-y^{2m}=\left(x^n\right)^2-\left(y^m\right)^2=\underline{\underline{\left(x^n+y^m\right)\left(x^n-y^m\right)}}$

106. c) $\dfrac{(-2x)^n}{-2x^n}=\dfrac{(-2)^n x^n}{(-2)x^n}=\underline{\underline{(-2)^{n-1}}}$

f) $\dfrac{a^{2n}-1}{a^n-1}=\dfrac{\overbrace{\left(a^n\right)^2-1}^{\text{3. Binom}}}{a^n-1}=\dfrac{\left(a^n+1\right)\cancel{\left(a^n-1\right)}}{\cancel{a^n-1}}=\underline{\underline{a^n+1}}$

g) $\dfrac{b^{4m}-b^{3m}}{b^{3m}-b^{2m}}=\dfrac{b^{3m}\cancel{\left(b^m-1\right)}}{b^{2m}\cancel{\left(b^m-1\right)}}=b^{3m-2m}=\underline{\underline{b^m}}$

i) $\dfrac{e^{3k}-e^k}{e^{2k-1}+e^{k-1}}=\dfrac{e^k\left(e^{2k}-1\right)}{\underbrace{e^{k-1}\left(e^k+1\right)}_{\substack{\text{weil}\\ e^k\cdot e^k\cdot e^{-1}+e^k\cdot e^{-1}}}}=\dfrac{e^{k-k+1}\left(e^{2k}-1\right)}{e^k+1}=$

$\dfrac{e\overbrace{\left[\left(e^k\right)^2-1\right]}^{\text{3. Binom}}}{e^k+1}=\dfrac{e\cancel{\left(e^k+1\right)}\left(e^k-1\right)}{\cancel{e^k+1}}=\underline{\underline{e\left(e^k-1\right)}}$

109. a) $\left(\dfrac{t+2}{t}\right)^2\cdot\left[\dfrac{1}{t}-\left(\dfrac{t-2}{2}\right)^{-1}\right]^{-2}=$

$\dfrac{(t+2)^2}{t^2}\cdot\left[\dfrac{1}{t}-\dfrac{2}{t-2}\right]^{-2}=$

$\dfrac{(t+2)^2}{t^2}\cdot\left[\dfrac{1}{t}\cdot\dfrac{t-2}{t-2}-\dfrac{2}{t-2}\cdot\dfrac{t}{t}\right]^{-2}=$

$\dfrac{(t+2)^2}{t^2}\cdot\left[\dfrac{t-2-2t}{t(t-2)}\right]^{-2}=$

$$\frac{(t+2)^2}{t^2}\cdot\frac{t^2\cdot(t-2)^2}{\underbrace{\left[-(t+2)\right]^2}_{(-1)^2(t+2)^2}}=\underline{\underline{(t-2)^2}}$$

b) $$2^{-2}\cdot(-b)^{6}_{\text{gerade}}-\left[4^3\left(-\frac{b}{2}\right)^{6}_{\text{gerade}}\right]+(-2)^{11}_{\text{ungerade}}\cdot(-1)^3\cdot 4^{-6}\cdot b^6=$$

$$2^{-2}\cdot b^6-4^3\cdot\frac{b^6}{2^6}+\underbrace{\left(-2^{11}\right)\cdot(-1)}_{(-)\cdot(-)=(+)}\cdot 4^{-6}\cdot b^6=$$

$$b^6\left[2^{-2}-\frac{4^3}{2^6}+2^{11}\cdot 4^{-6}\right]=$$

$$b^6\left[2^{-2}-\frac{\left(2^2\right)^3}{2^6}+2^{11}\cdot\left(2^2\right)^{-6}\right]=$$

$$b^6\left[2^{-2}-1+2^{11}\cdot 2^{-12}\right]=$$

$$b^6\left[2^{-2}-1+2^{-1}\right]=$$

$$b^6\left[\frac{1}{4}-\frac{4}{4}+\frac{2}{4}\right]=b^6\left[-\frac{1}{4}\right]=\underline{\underline{-\frac{b^6}{4}}}$$

c) $$\frac{\frac{2}{3(u-1)}}{\frac{1}{3^2(u-v)^2}}:\left[1+\frac{1}{u}-\frac{2}{v}+\frac{\left(1-\frac{1}{v}\right)^2}{\frac{1-u}{u}}\right]=$$

$$\left[\frac{2}{3(u-1)}\cdot\frac{3^2(u-v)^2}{1}\right]:\left[1+\frac{1}{u}-\frac{2}{v}+\frac{\left(\frac{v-1}{v}\right)^2}{\frac{1-u}{u}}\right]=$$

$$\frac{6(u-v)^2}{(u-1)}:\left[1+\frac{1}{u}-\frac{2}{v}+\frac{(v-1)^2}{v^2}\cdot\frac{u}{(1-u)}\right]=$$

$$\frac{6(u-v)^2}{(u-1)}:\left[\frac{uv^2(1-u)}{uv^2(1-u)}+\frac{1}{u}\cdot\frac{v^2(1-u)}{v^2(1-u)}-\frac{2}{v}\cdot\frac{uv(1-u)}{uv(1-u)}+\frac{(v-1)^2}{v^2}\cdot\frac{u}{(1-u)}\cdot\frac{u}{u}\right]=$$

$$\frac{6(u-v)^2}{(u-1)} \cdot \frac{uv^2(1-u)}{uv^2-u^2v^2+v^2-uv^2-2uv+2u^2v+u^2(v^2-2v+1)} =$$

$$\frac{6(u-v)^2}{(u-1)} \cdot \frac{-uv^2(u-1)}{\cancel{uv^2}\cancel{-u^2v^2}+v^2\cancel{-uv^2}-2uv\cancel{+2u^2v}\cancel{+u^2v^2}\cancel{-2u^2v}+u^2} =$$

$$\frac{6\cancel{(u-v)^2}}{\cancel{(u-1)}} \cdot \frac{-uv^2\cancel{(u-1)}}{\cancel{v^2-2uv+u^2}} = \underline{\underline{-6uv^2}}$$

d) $$\left[\frac{1}{p^2}+\frac{1}{q^2}+\frac{2}{pq}-\frac{r^2}{p^2q^2}\right]:\left[\left(\frac{1}{p}+\frac{1}{q}-\frac{r}{pq}\right)(p+q+r)\right]=$$

$$\left[\frac{1}{p^2}\cdot\frac{q^2}{q^2}+\frac{1}{q^2}\cdot\frac{p^2}{p^2}+\frac{2}{pq}\cdot\frac{pq}{pq}-\frac{r^2}{p^2q^2}\right]:\left[\left(\frac{1}{p}\cdot\frac{q}{q}+\frac{1}{q}\cdot\frac{p}{p}-\frac{r}{pq}\right)(p+q+r)\right]=$$

$$\left[\frac{\overbrace{q^2+p^2+2pq}^{\text{Binom}}-r^2}{p^2q^2}\right]:\left[\frac{q+p-r}{pq}\cdot\frac{p+q+r}{1}\right]=$$

$$\frac{\overbrace{(p+q)^2-r^2}^{\text{3. Binom}}}{p^2q^2}\cdot\frac{pq}{(p+q-r)(p+q+r)}=$$

$$\frac{\cancel{(p+q+r)(p+q-r)}}{p^2q^2}\cdot\frac{pq}{\cancel{(p+q-r)(p+q+r)}}=\frac{pq}{p^2q^2}=\underline{\underline{\frac{1}{pq}}}$$