8 Logarithmieren

8.16 Übungen Frommenwiler

371. d) $\ln(2-y)=-2$ |Definition $D=\{y\in\mathbf{R}\,|\,y<2\}$

$\ln(2-y)=-2 \quad\Leftrightarrow\quad e^{-2}=2-y$ $|+y-e^{-2}$

$y=2-e^{-2}=\underline{1.8647}$

Kontrolle: $\ln\left(2-2+e^{-2}\right)=-2$

$\ln\left(e^{-2}\right)=-2$ (w)

$L=\underline{\underline{\{2-e^{-2}\}}}$

e) $4.2+\ln\left(x^3\right)=7.6$ $|-4.2$ $D=\{x\in\mathbf{R}\,|\,x>0\}$

$\ln\left(x^3\right)=3.4$ |Definition

$\ln\left(x^3\right)=3.4 \quad\Leftrightarrow\quad e^{3.4}=x^3$ $|\sqrt[3]{\;}$

$x=\sqrt[3]{e^{3.4}}=\underline{e^{\frac{3.4}{3}}}=\underline{e^{\frac{17}{15}}}$

Kontrolle: $4.2+\ln\left(\left(e^{\frac{3.4}{3}}\right)^3\right)=7.6$

$4.2+\ln\left(e^{3.4}\right)=7.6$ (w)

$L=\underline{\underline{\left\{e^{\frac{17}{15}}\right\}}}$

f) $1.8-\ln\sqrt[3]{2t}=2.1$ $|-1.8$ $D=\{t\in\mathbf{R}\,|\,t>0\}$

$-\ln\sqrt[3]{2t}=0.3$ |TU

$\ln(2t)^{-\frac{1}{3}}=\frac{3}{10} \quad\Leftrightarrow\quad e^{\frac{3}{10}}=(2t)^{-\frac{1}{3}}$ $|(\;)^{-3}$

$e^{-\frac{9}{10}} = 2t \quad |:2$

$t = \frac{1}{2} \cdot e^{-\frac{9}{10}} = \frac{1}{2 \cdot e^{\frac{9}{10}}} = \frac{1}{2 \cdot \sqrt[10]{e^9}}$

Kontrolle: $1.8 - \ln \sqrt[3]{2 \cdot \frac{1}{2} \cdot e^{-\frac{9}{10}}} = 2.1$

$1.8 - \underbrace{\ln\left(e^{-\frac{9}{10 \cdot 3}}\right)}_{-0.3} = 2.1 \quad (w)$

$$L = \left\{ \frac{e^{-\frac{9}{10}}}{2} \right\}$$

372. c) $\lg\left(5^x\right) = \lg\left(2^x\right) + 2 \quad \left| -\lg\left(2^x\right) \right. \quad D = \mathbb{R}$

$\lg\left(5^x\right) - \lg\left(2^x\right) = 2 \quad |$Divisionsregel

$\lg\left(\frac{5}{2}\right)^x = 2 \quad |$Potenzregel

$x \cdot \lg\left(\frac{5}{2}\right) = 2 \quad \left| : \lg\left(\frac{5}{2}\right) \right.$

$x = \frac{2}{\lg(2.5)} = \underline{5.03}$

Kontrolle: $\underbrace{\lg\left(5^{5.03}\right)}_{3.5129} = \underbrace{\lg\left(2^{5.03}\right) + 2}_{3.5129} \quad (w)$

$$L = \left\{ \frac{2}{\lg(2.5)} \right\}$$

d) $\ln\left(10^y\right) - \ln\left(5^y\right) = -1 \quad |$Divisionsregel $\quad D = \mathbb{R}$

$\ln\left(\frac{10}{5}\right)^y = -1 \quad |$Potenzregel

$y \cdot \ln(2) = -1 \quad |: \ln(2)$

$$y = \frac{-1}{\underline{\ln(2)}} = \underline{-1.44}$$

Kontrolle: $\underbrace{\ln\left(10^{-1.44}\right) - \ln\left(5^{-1.44}\right)}_{-1} = -1$ (w)

$$L = \underline{\underline{\left\{\frac{-1}{\ln(2)}\right\}}}$$

374. a) $2 \cdot \lg x = \lg(2x)$ |Potenzregel $D = \{x \in \mathbf{R} \mid x > 0\}$

$\lg\left(x^2\right) = \lg(2x)$ |Numeri gleichsetzen

$x^2 = 2x$ |$-2x$

$x^2 - 2x = 0$ |faktorisieren

$x(x-2) = 0$

$x_1 = \underline{0} \notin D \quad \vee \quad x = \underline{2} \in D$

Kontrolle: $\underset{0.6021}{2 \cdot \lg 2} = \underbrace{\lg(2 \cdot 2)}_{0.6021}$ (w)

$$L = \underline{\underline{\{2\}}}$$

c) $\ln y + \ln(y+4) = 2 \cdot \ln(y+1)$ |Produkt-u. Potenzregel $D = \{y \in \mathbf{R} \mid y > 0\}$

$\ln\left(y^2 + 4y\right) = \ln(y+1)^2$ |Numeri gleichsetzen

$y^2 + 4y = y^2 + 2y + 1$ |$-y^2 - 2y$

$2y = 1$

$y = \frac{1}{\underline{2}}$

Kontrolle: $\underbrace{\ln\left(\frac{1}{2}\right) + \ln\left(\frac{1}{2} + 4\right)}_{0.8109} = \underbrace{2 \cdot \ln\left(\frac{1}{2} + 1\right)}_{0.8109}$ (w)

$$L = \underline{\underline{\left\{\frac{1}{2}\right\}}}$$

e) $\frac{\lg(x+1)}{\lg(x-1)} = 2$ $\quad |\cdot \lg(x-1) \qquad D = \{x \in \mathbb{R} \,|\, x > 1 \;\wedge\; x \neq 2\}$

$\lg(x+1) = 2 \cdot \lg(x-1)$ $\quad |$Potenzregel

$\lg(x+1) = \lg(x-1)^2$ $\quad |$Numeri gleichsetzen

$x + 1 = x^2 - 2x + 1$ $\quad |-x-1$

$0 = x^2 - 3x = x(x-3)$

$x_1 = \underline{0} \notin D \quad \vee \quad x_2 = \underline{3}$

Kontrolle: $\underbrace{\frac{\lg(3+1)}{\lg(3-1)}}_{2} = 2 \qquad (w)$

$L = \underline{\underline{\{3\}}}$

375. a) $\lg(2a) - \lg(a^2) = 0.1$ $\quad |$Divisionsregel $\qquad D = \{a \in \mathbb{R} \,|\, a > 0\}$

$\lg\left(\frac{2a}{a^2}\right) = 0.1$ $\quad |$Definition

$\lg\left(\frac{2a}{a^2}\right) = 0.1 \quad \Leftrightarrow \quad 10^{0.1} = \frac{2a}{a^2}$ $\quad |\cdot a^2$

$a^2 \cdot 10^{0.1} = 2a$ $\quad |-2a$

$a^2 \cdot 10^{0.1} - 2a = 0$ $\quad |$faktorisieren

$a(a \cdot 10^{0.1} - 2) = 0$

$a_1 = 0 \notin D \quad \vee \quad a_2 = \underline{\frac{2}{10^{0.1}}} = \underline{\frac{2}{\sqrt[10]{10}}} = \underline{1.59}$

Kontrolle: $\underbrace{\lg(2 \cdot 1.59) - \lg(1.59^2)}_{0.1} = 0.1 \qquad (w)$

$L = \underline{\underline{\left\{\frac{2}{\sqrt[10]{10}}\right\}}}$

c) $\ln(2t+1)-\ln(t)=1$ |Divisionsregel $D=\{t\in\mathbf{R}\,|\,t>0\}$

$\ln\left(\frac{2t+1}{t}\right)=1$ |Definition

$\ln\left(\frac{2t+1}{t}\right)=1 \quad\Leftrightarrow\quad e^1=\frac{2t+1}{t}$ $|\cdot t$

$e\cdot t=2t+1$ $|-2t$

$e\cdot t-2t=1$ |faktorisieren

$t(e-2)=1$ $|:(e-2)$

$t=\frac{1}{e-2}=\underline{1.39}$

Kontrolle: $\underbrace{\ln(2\cdot 1.39+1)-\ln(1.39)}_{1}=1$ (w)

$L=\underline{\underline{\left\{\frac{1}{e-2}\right\}}}$

h) $x^{\lg x}=10$ |lg $D=\{x\in\mathbf{R}\,|\,x>0\}$

$\lg\left(x^{\lg x}\right)=\lg \underset{1}{10}$ |Potenzregel

$\lg x\cdot\lg x=1$

$(\lg x)^2=1$ $|\sqrt{}$

$\lg x_{1,2}=\pm 1$ |Definition

$\lg x_1=1 \quad\Leftrightarrow\quad \underline{10^1}=x_1$

$\lg x_2=-1 \quad\Leftrightarrow\quad \underline{10^{-1}}=x_2$

Kontrolle: $10^{\overset{\lg 10}{1}}=10$ (w)

$\left(10^{-1}\right)^{\underbrace{\lg 10^{-1}}_{-1}}=10$ (w)

$L=\underline{\underline{\left\{\frac{1}{10};\,10\right\}}}$

376. a) $5^{\ln x} = 2 \cdot 3^{\ln x}$ sei $u = \ln x$ $D = \{x \in \mathbb{R} \mid x > 0\}$

Substitution:

$$5^{u} = 2 \cdot 3^{u} \qquad |:3^{u}$$

$$(5/3)^{u} = 2 \qquad |\ln$$

$$u \cdot \ln(5/3) = \ln 2 \qquad |:\ln(5/3)$$

$$u = \underline{\frac{\ln 2}{\ln(5/3)}}$$

Rücksubstitution:

$$\frac{\ln 2}{\ln(5/3)} = \ln x \qquad |\text{Definition}$$

$$\ln x = \frac{\ln 2}{\ln(5/3)} \quad \Leftrightarrow \quad \underline{e^{\frac{\ln 2}{\ln(5/3)}}} = x$$

$$x = \left(e^{\ln 2}\right)^{\frac{1}{\ln(5/3)}} = \underline{2^{\frac{1}{\ln(5/3)}}} = \underline{3.88}$$

Kontrolle:

$$\underbrace{5^{\ln 3.88}}_{8.8806} = \underbrace{2 \cdot 3^{\ln 3.88}}_{8.8806} \qquad (w)$$

somit:

$$L = \underline{\underline{\left\{2^{\frac{1}{\ln(5/3)}}\right\}}}$$

c) $\lg\sqrt{p} = \sqrt{\lg p} \quad \rightarrow \quad \frac{1}{2} \cdot \lg p = (\lg p)^{\frac{1}{2}}$ $D = \{p \in \mathbb{R} \mid p > 0\}$

sei $u = \lg p$

Substitution:

$$\frac{1}{2} \cdot u = u^{\frac{1}{2}} \qquad |(\;)^{2}$$

$$\frac{u^{2}}{4} = u \qquad |\cdot 4$$

$$u^{2} = 4u \qquad |-4u$$

$$u^{2} - 4u = 0 \qquad |\text{faktorisieren}$$

$$u \cdot (u - 4) = 0$$

$$u_1 = \underline{0} \quad \vee \quad u_2 = \underline{4}$$

Rücksubstitution:

$$0 = \lg p \quad \Leftrightarrow \quad 10^{0} = p$$

$$4 = \lg p \quad \Leftrightarrow \quad 10^{4} = p$$

Kontrolle: $p = 1$: $\underbrace{\lg\sqrt{1}}_{0} = \underbrace{\sqrt{\lg 1}}_{0}$ (w)

$p = 10^4$: $\underbrace{\lg\sqrt{10^4}}_{2} = \underbrace{\sqrt{\lg 10^4}}_{2}$ (w)

somit: $L = \{1;\ 10^4\}$

d) $\lg\sqrt{u} = \frac{1}{2} + \frac{1}{\lg u} \rightarrow \frac{1}{2}\cdot\lg u = \frac{1}{2} + \frac{1}{\lg u}$ $D = \{u \in \mathbf{R} \mid u > 0\}$

sei $v = \lg u$

Substitution:

$\frac{1}{2}\cdot v = \frac{1}{2} + \frac{1}{v}$ $|\cdot 2v$

$v^2 = v + 2$ |ordnen

$v^2 - v - 2 = 0$ |faktorisieren

$(v-2)(v+1) = 0$

$v_1 = 2 \quad \vee \quad v_2 = -1$

Rücksubstitution: $2 = \lg u \Leftrightarrow 10^2 = u$

$-1 = \lg u \Leftrightarrow 10^{-1} = u$

Kontrolle: $u = 100$: $\lg\sqrt{100} = \frac{1}{2} + \frac{1}{\lg 100}$ (w)

$u = 10^{-1}$: $\lg\sqrt{10^{-1}} = \frac{1}{2} + \frac{1}{\lg 10^{-1}}$ (w)

somit: $L = \{10^{-1};\ 10^2\}$

377. b) $n\cdot\lg q = \lg K_n - \lg K_0$ $|+\lg K_0$

$n\cdot\lg q + \lg K_0 = \lg K_n$ |Potenz- und Produktregel

$\lg(q^n\cdot K_0) = \lg(K_n)$ |Numeri gleichsetzen

$K_n = K_0\cdot q^n$

c) $t=\frac{\lg m_0-\lg m}{\lg 2}\cdot T$ $\quad|\cdot\lg 2 \quad |:T$

$\frac{t}{T}\cdot\lg 2=\lg m_0-\lg m$ $\quad|-\lg m_0$

$\frac{t}{T}\cdot\lg 2-\lg m_0=-\lg m$ $\quad|\cdot(-1)$

$\lg m_0-\frac{t}{T}\cdot\lg 2=\lg m$ $\quad$|Potenz- und Divisionsregel

$\lg\left(\frac{m_0}{2^{\frac{t}{T}}}\right)=\lg(m)$ $\quad$|Numeri gleichsetzen

$m=\underline{\frac{m_0}{2^{\frac{t}{T}}}}=\underline{m_0\cdot 2^{-\frac{t}{T}}}$

d) $t=a+\frac{\ln N-\ln N_0}{\ln n}$ $\quad|\cdot\ln n$

$t\cdot\ln n=a\cdot\ln n+\ln N-\ln N_0$ $\quad|-a\cdot\ln n$

$t\cdot\ln n-a\cdot\ln n=\ln N-\ln N_0$ $\quad$|faktorisieren und Divisionsregel

$\ln n\cdot(t-a)=\ln\left(\frac{N}{N_0}\right)$ $\quad|:(t-a)$

$\ln n=\frac{\ln\left(\frac{N}{N_0}\right)}{t-a}$ $\quad$|Definition

$e^{\frac{\ln\left(\frac{N}{N_0}\right)}{t-a}}=\left[e^{\ln\left(\frac{N}{N_0}\right)}\right]^{\frac{1}{t-a}}=n$ $\quad|e^{\ln\square}=\square$

$n=\underline{\underline{\left(\frac{N}{N_0}\right)^{\frac{1}{t-a}}}}$

378. a) $\ln x - \ln a = b \quad |+\ln a$

$\ln x = b + \ln a \quad |\ln e^{b} = b$

$\ln(x) = \ln e^{b} + \ln a = \ln(e^{b} \cdot a) \quad |\text{Numeri gleichsetzen}$

$x = \underline{a \cdot e^{b}}$

$L = \underline{\underline{\{x \mid x = a \cdot e^{b}\}}}$

b) $\lg(x-a) = \lg x - \lg a \quad |\text{Divisionsregel}$

$\lg(x-a) = \lg\left(\frac{x}{a}\right) \quad |\text{Numeri gleichsetzen}$

$x - a = \frac{x}{a} \quad |\cdot a$

$ax - a^{2} = x \quad |+a^{2} - x$

$ax - x = a^{2} \quad |\text{faktorisieren}$

$x(a-1) = a^{2} \quad |:(a-1)$

$x = \underline{\frac{a^{2}}{a-1}}$

$L = \underline{\underline{\left\{x \,\middle|\, x = \frac{a^{2}}{a-1}\right\}}}$