

## 9 Gleichungen

### 9.17 Übungen Frommenwiler (noch korrigieren)

$$324. \text{ a) } \frac{45}{2y-9} - 2 = -\frac{27}{9-2y} \cdot \frac{-1}{-1}$$

$$2y-9 \neq 0 \rightarrow 2y \neq 9 \rightarrow y \neq \frac{9}{2} \rightarrow D = \mathbb{R} \setminus \left\{ \frac{9}{2} \right\}$$

$$\frac{45}{2y-9} - 2 = \frac{27}{2y-9}$$

$$| -\frac{27}{2y-9} + 2$$

$$\frac{45}{2y-9} - \frac{27}{2y-9} = 2$$

$$\frac{45-27}{2y-9} = 2$$

$$| \cdot (2y-9)$$

$$18 = 2 \cdot (2y-9)$$

$$18 = 4y - 18$$

$$| +18$$

$$36 = 4y$$

$$| :4$$

$$y = \underline{\underline{9}} \in D$$

$$\text{Kontrolle: } \frac{45}{\underbrace{2 \cdot 9 - 9}_3} - 2 = -\frac{27}{\underbrace{9 - 2 \cdot 9}_3} \quad (\text{w})$$

$$\text{somit: } L = \underline{\underline{\{9\}}}$$

$$b) \frac{6x+5}{2} + \frac{4x-13}{11x+1} = 3x$$

$$11x+1 \neq 0 \rightarrow 11x \neq -1 \rightarrow x \neq -\frac{1}{11} \rightarrow D = \mathbb{R} \setminus \left\{ -\frac{1}{11} \right\}$$

$$\frac{6x+5}{2} \cdot \frac{11x+1}{11x+1} + \frac{4x-13}{11x+1} \cdot \frac{2}{2} = 3x \cdot \frac{11x+1}{11x+1} \cdot \frac{2}{2} \quad | \cdot \text{HN}$$

$$66x^2 + 6x + 55x + 5 + 8x - 26 = 66x^2 + 6x \quad | -66x^2$$

$$69x - 21 = 6x \quad | -6x + 21$$

$$63x = 21 \quad | : 63$$

$$x = \frac{21}{63} = \frac{1}{3} \in D$$

$$\text{Kontrolle: } \underbrace{\frac{6 \cdot \frac{1}{3} + 5}{2} + \frac{4 \cdot \frac{1}{3} - 13}{11 \cdot \frac{1}{3} + 1}}_1 = 3 \cdot \frac{1}{3} \quad (\text{w})$$

$$\text{somit: } L = \underline{\underline{\left\{ \frac{1}{3} \right\}}}$$

$$c) \frac{4z+6}{2z-10} - \frac{6z-43}{5-z} = 124$$

$$5-z \neq 0 \rightarrow z \neq 5 \rightarrow D = \mathbb{R} \setminus \{5\}$$

$$\frac{2(2z+3)}{-2(5-z)} - \frac{6z-43}{5-z} = 124 \quad | \text{kürzen}$$

$$\frac{-(2z+3) - 6z + 43}{5-z} = 124 \quad | \cdot (5-z)$$

$$-2z - 3 - 6z + 43 = 124(5-z) = 620 - 124z \quad | \text{zusammenfassen}$$

$$-8z + 40 = 620 - 124z \quad | +124z - 40$$

$$116z = 580 \quad | : 116$$

$$z = \frac{580}{116} = 5 \notin D$$

$$\text{somit: } L = \underline{\underline{\{ \}}}$$

326. a)  $\frac{1}{x+1} + \frac{x}{x+2} + \frac{\cancel{x+3}}{\underbrace{(x+3)^2}} = 1$  | :HN  
weil  $x = -3$  keine gültige Lösung ist!

$D = \mathbb{R} \setminus \{-1; -2; -3\}$

$(x+2)(x+3) + x(x+1)(x+3) + (x+1)(x+2) = (x+1)(x+2)(x+3)$

$x^2 + 5x + 6 + x(x^2 + 4x + 3) + x^2 + 3x + 2 = (x+1)(x^2 + 5x + 6)$

$x^2 + 5x + 6 + x^3 + 4x^2 + 3x + x^2 + 3x + 2 = x^3 + 5x^2 + 6x + x^2 + 5x + 6$  | ordnen

$x^3 + 6x^2 + 11x + 8 = x^3 + 6x^2 + 11x + 6$  |  $-x^3 - 6x^2 - 11x$

$8 = 6$  (f)

$L = \{ \}$

b)  $\frac{1-4t}{(t-2)(t-1)} - \frac{3(t-1)}{(t-6)(t-2)} + \frac{7\cancel{t}}{\cancel{t}(t-6)} = 0$  | :HN

$D = \mathbb{R} \setminus \{0; 1; 2; 6\}$

$(1-4t)(t-6) - 3(t-1)^2 + 7(t-1)(t-2) = 0$  | ausmult.

$(25t - 6 - 4t^2) - 3(t^2 - 2t + 1) + 7(t^2 - 3t + 2) = 0$

$25t - 6 - 4t^2 - 3t^2 + 6t - 3 + 7t^2 - 21t + 14 = 0$

$10t + 5 = 0$  | :5

$2t + 1 = 0$  | -1

$2t = -1$  | :2

$t = -\frac{1}{2} \in D$

Kontrolle:  $\frac{1-4\left(-\frac{1}{2}\right)}{\left(-\frac{1}{2}\right)^2 - 3\left(-\frac{1}{2}\right) + 2} - \frac{3\left(-\frac{1}{2}-1\right)}{\left(-\frac{1}{2}\right)^2 - 8\left(-\frac{1}{2}\right) + 12} + \frac{7\left(-\frac{1}{2}\right)}{\left(-\frac{1}{2}\right)^2 - 6\left(-\frac{1}{2}\right)} = 0$  (w)  
mit Taschenrechner: 0

somit:  $L = \left\{ -\frac{1}{2} \right\}$

$$c) \quad \frac{3x+4}{x-5} - \frac{x-9}{x-7} = \frac{2x^2-13x+27}{(x-5)(x-7)} \quad | \cdot \text{HN}$$

$$D = \mathbf{R} \setminus \{5; 7\}$$

$$(3x+4)(x-7) - (x-9)(x-5) = 2x^2 - 13x + 27 \quad | \text{ausmult.}$$

$$3x^2 - 17x - 28 - (x^2 - 14x + 45) = 2x^2 - 13x + 27$$

$$3x^2 - 17x - 28 - x^2 + 14x - 45 = 2x^2 - 13x + 27$$

$$2x^2 - 3x - 73 = 2x^2 - 13x + 27 \quad | -2x^2 + 13x + 73$$

$$10x = 100$$

$$x = \underline{10} \in D$$

$$\text{Kontrolle: } \frac{\underbrace{3 \cdot 10 + 4}_{\frac{97}{15}}}{\underbrace{10 - 5}_{\frac{97}{15}}} - \frac{\underbrace{10 - 9}_{\frac{97}{15}}}{\underbrace{10 - 7}_{\frac{97}{15}}} = \frac{\underbrace{2 \cdot 10^2 - 13 \cdot 10 + 27}_{\frac{97}{15}}}{\underbrace{10^2 - 12 \cdot 10 + 35}_{\frac{97}{15}}} \quad (w)$$

$$\text{somit: } L = \underline{\underline{\{10\}}}$$

$$d) \frac{3}{z+3} - \frac{3}{z+8} = \frac{5}{z-8} - \frac{5}{z-5}$$

|gleichn.

$$D = \mathbf{R} \setminus \{-8; -3; 5; 8\}$$

$$\frac{3(z+8) - 3(z+3)}{(z+3)(z+8)} = \frac{5(z-5) - 5(z-8)}{(z-8)(z-5)}$$

|ausmult.

$$\frac{3z + 24 - 3z - 9}{(z+3)(z+8)} = \frac{5z - 25 - 5z + 40}{(z-8)(z-5)}$$

|zusammenf.

$$\frac{15}{(z+3)(z+8)} = \frac{15}{(z-8)(z-5)}$$

| $N_1 = N_2$ 

$$(z+3)(z+8) = (z-8)(z-5)$$

$$z^2 + 11z + 24 = z^2 - 13z + 40$$

| $-z^2 + 13z - 24$ 

$$24z = 16$$

|: 24

$$z = \frac{16}{24} = \frac{2}{3} \in D$$

$$\text{Kontrolle: } \frac{\frac{2}{3}}{\frac{2}{3}+3} - \frac{\frac{2}{3}}{\frac{2}{3}+8} = \frac{\frac{2}{3}}{\frac{2}{3}-8} - \frac{\frac{2}{3}}{\frac{2}{3}-5} \quad (w)$$

$$\text{somit: } L = \left\{ \frac{2}{3} \right\}$$

oder

$$\frac{3}{z+3} - \frac{3}{z+8} = \frac{5}{z-8} - \frac{5}{z-5}$$

|auskl.|gleichn.

$$D = \mathbf{R} \setminus \{-8; -3; 5; 8\}$$

$$\frac{3(z+8 - z - 3)}{(z+3)(z+8)} = \frac{5(z-5 - z + 8)}{(z-8)(z-5)}$$

|zusammenf.

$$\frac{3 \cdot 5}{(z+3)(z+8)} = \frac{5 \cdot 3}{(z-8)(z-5)}$$

| $N_1 = N_2$ 

$$(z+3)(z+8) = (z-8)(z-5)$$

Fortsetzung siehe oben!

$$e) \frac{3}{1+\frac{1}{p}} \cdot \frac{p}{p} - \frac{1+\frac{1}{p}}{1-\frac{1}{p}} \cdot \frac{p}{p} = 2$$

|Brüche weg

$$D = \mathbf{R} \setminus \{-1; 0; 1\}$$

$$\frac{3p}{p+1} - \frac{p+1}{p-1} = 2$$

|·HN

$$3p(p-1) - (p+1)(p+1) = 2(p+1)(p-1)$$

|ausmulti.

$$3p^2 - 3p - (p^2 + 2p + 1) = 2(p^2 - 1)$$

$$3p^2 - 3p - p^2 - 2p - 1 = 2p^2 - 2$$

|zusammenf.

$$2p^2 - 5p - 1 = 2p^2 - 2$$

| $-2p^2 + 1$ 

$$-5p = -1$$

| $:(-5)$ 

$$p = \frac{-1}{-5} = \frac{1}{5} \in D$$

$$\text{Kontrolle: } \underbrace{\frac{3}{1+5} - \frac{1+5}{1-5}}_2 = 2 \text{ (w)}$$

$$\text{somit: } L = \underline{\underline{\left\{ \frac{1}{5} \right\}}}$$

$$f) \frac{\frac{2}{0.9} - \frac{2}{y}}{\frac{1}{0.9} + \frac{1}{y}} = \frac{2}{3}$$

$$y \neq 0 \wedge \frac{10}{9} + \frac{1}{y} \neq 0 \rightarrow \frac{1}{y} \neq -\frac{10}{9} \rightarrow y \neq -\frac{9}{10} \rightarrow D = \mathbb{R} \setminus \left\{ -\frac{9}{10}; 0 \right\}$$

$$\frac{2 \cdot 10}{9} - \frac{2}{y} \cdot \frac{9y}{9y} = \frac{2}{3} \quad | \text{Brüche weg}$$

$$\frac{1 \cdot 10}{9} + \frac{1}{y} \cdot \frac{9y}{9y} = \frac{2}{3}$$

$$\frac{20y - 18}{10y + 9} = \frac{2}{3}$$

$$3(20y - 18) = 2(10y + 9) \quad | \text{ausmulti.}$$

$$60y - 54 = 20y + 18 \quad | -20y + 54$$

$$40y = 72 \quad | : 40$$

$$y = \frac{72}{40} = \frac{9}{5} \in D$$

$$\text{Kontrolle: } \frac{\frac{2}{0.9} - \frac{2 \cdot 5}{9}}{\frac{1}{0.9} + \frac{1}{9}} = \frac{2}{3} \quad (w)$$

$$\text{somit: } L = \left\{ \frac{9}{5} \right\}$$