13 Wurzeln; Potenzen mit rationalen Exponenten

13.1 Der allgemeine Wurzelbegriff

1. a) 3 b) 5 c) 2 d) $\frac{1}{2}$ e) $\frac{1}{3}$

2. a) 0 b) $\frac{1}{2}$ c) 1 d) $\frac{1}{2}$ e) 0

3. a) 10 b) 0,1 c) 10 d) 0,1 e) 0,1

4. a) $|a|$ b) a^2 c) $|a^3|$ d) $|a|$ e) a^2

5. a) $a^{\frac{1}{2}}$ b) $b^{\frac{1}{3}}$ c) $c^{\frac{1}{4}}$ d) $x^{\frac{1}{5}}$ e) $(x+y)^{\frac{1}{2}}$

6. a) $a^{\frac{2}{3}}$ b) $b^{\frac{3}{4}}$ c) $c^{\frac{2}{5}}$ d) $x^{\frac{4}{3}}$ e) $y^{\frac{7}{4}}$

7. a) $a^{-\frac{1}{4}}$ b) $b^{-\frac{1}{3}}$ c) $c^{-\frac{2}{3}}$ d) $x^{-\frac{4}{5}}$ e) $y^{-\frac{5}{4}}$

8. a) $\sqrt[3]{a}$ b) $\sqrt[3]{b^2}$ c) $\sqrt[4]{c^3}$ d) $\sqrt[6]{x^5}$ e) $\sqrt[5]{y^6} = y\sqrt[5]{y}$

9. a) $\frac{1}{\sqrt{a}}$ b) $\frac{1}{\sqrt[4]{b}}$ c) $\frac{1}{\sqrt[5]{c^4}}$ d) $\frac{1}{\sqrt[4]{x^3}}$ e) $\frac{1}{\sqrt[3]{y^4}}$

10. a) $\sqrt{3}$ b) $\sqrt[3]{5}$ c) $\frac{1}{\sqrt{7}}$ d) $\frac{1}{\sqrt[4]{6}}$ e) $\frac{1}{\sqrt[3]{10}}$

11. a) $\sqrt{9} = 3$ b) $\sqrt[3]{8} = 2$ c) $\sqrt[4]{81} = 3$ d) $\sqrt[5]{32} = 2$ e) $\sqrt[6]{1} = 1$

12. a) $\frac{1}{\sqrt{4}} = \frac{1}{2}$ b) $\frac{1}{\sqrt[3]{27}} = \frac{1}{3}$ c) $\frac{1}{\sqrt[4]{16}} = \frac{1}{2}$ d) $\frac{1}{\sqrt[5]{1}} = 1$ e) $\sqrt[7]{0} = 0$

13. a) $\sqrt[3]{1^2} = 1$ b) $\sqrt[3]{8^2} = \sqrt[3]{64} = 4$ c) $\sqrt[4]{0^3} = 0$

d) $\sqrt{100^3} = \sqrt{1\,000\,000} = 1\,000$ e) $\frac{1}{\sqrt[5]{1^4}} = 1$

14. a) $\sqrt{\frac{9}{16}} = \frac{3}{4}$ b) $\sqrt[3]{\frac{8}{27}} = \frac{2}{3}$ c) $\sqrt[4]{\frac{1}{81}} = \frac{1}{3}$ d) $\frac{1}{\sqrt{\frac{1}{4}}} = 1 : \frac{1}{2} = 2$

e) $\frac{1}{\sqrt[4]{\frac{1}{16}}} = 1 : \frac{1}{2} = 2$

13.2 Rechnen mit Potenzen mit rationalen Exponenten

1. a) $a^{\frac{1}{2}} \cdot a^{\frac{1}{3}} = a^{\frac{5}{6}} = \sqrt[6]{a^5}$ b) $b^{\frac{1}{2}} \cdot b^{\frac{1}{4}} = b^{\frac{3}{4}} = \sqrt[4]{b^3}$

c) $x^{\frac{1}{3}} \cdot x^{\frac{1}{4}} = x^{\frac{7}{12}} = \sqrt[12]{x^7}$ d) $y^{\frac{1}{3}} \cdot y^{\frac{1}{5}} = y^{\frac{8}{15}} = \sqrt[15]{y^8}$

e) $a^{\frac{1}{2}} \cdot a^{\frac{3}{4}} = a^{\frac{5}{4}} = \sqrt[4]{a^5} = a\sqrt[4]{a}$ f) $x^{\frac{2}{3}} \cdot x^{\frac{1}{4}} = x^{\frac{11}{12}} = \sqrt[12]{x^{11}}$

g) $y^{\frac{4}{5}} \cdot y^{\frac{1}{2}} = y^{\frac{13}{10}} = \sqrt[10]{y^{13}} = y\sqrt[10]{y^3}$ h) $a^{\frac{1}{3}} \cdot a^{\frac{3}{5}} = a^{\frac{14}{15}} = \sqrt[15]{a^{14}}$

2. a) $2^{\frac{1}{2}} \cdot 2^{\frac{1}{3}} = 2^{\frac{5}{6}} = \sqrt[6]{2^5}$
   b) $7^{\frac{1}{3}} \cdot 7^{\frac{1}{6}} = 7^{\frac{1}{2}} = \sqrt{7}$
   c) $3^{\frac{1}{4}} \cdot 3^{\frac{1}{2}} = 3^{\frac{3}{4}} = \sqrt[4]{3^3}$
   d) $5^{\frac{1}{3}} \cdot 5^{\frac{1}{5}} = 5^{\frac{8}{15}} = \sqrt[15]{5^8}$
   e) $3^{\frac{1}{2}} \cdot 3^{\frac{2}{3}} = 3^{\frac{7}{6}} = \sqrt[6]{3^7} = 3\sqrt[6]{3}$
   f) $5^{\frac{1}{3}} \cdot 5^{\frac{3}{4}} = 5^{\frac{13}{12}} = \sqrt[12]{5^{13}} = 5\sqrt[12]{5}$
   g) $2^{\frac{1}{2}} \cdot 2^{\frac{2}{5}} = 2^{\frac{9}{10}} = \sqrt[10]{2^9}$
   h) $3^{\frac{2}{3}} \cdot 3^{\frac{3}{4}} = 3^{\frac{17}{12}} = \sqrt[12]{3^{17}} = 3\sqrt[12]{3^5}$
3. a) $a^{\frac{1}{2}} : a^{\frac{1}{3}} = a^{\frac{1}{6}} = \sqrt[6]{a}$
   b) $b^{\frac{1}{2}} : b^{\frac{1}{5}} = b^{\frac{3}{10}} = \sqrt[10]{b^3}$
   c) $x^{\frac{1}{3}} : x^{\frac{1}{5}} = x^{\frac{2}{15}} = \sqrt[15]{x^2}$
   d) $y^{\frac{1}{3}} : y^{\frac{1}{4}} = y^{\frac{1}{12}} = \sqrt[12]{y}$
   e) $a^{\frac{2}{3}} : a^{\frac{1}{2}} = a^{\frac{1}{6}} = \sqrt[6]{a}$
   f) $x^{\frac{3}{4}} : x^{\frac{1}{2}} = x^{\frac{1}{4}} = \sqrt[4]{x}$
   g) $y^{\frac{4}{5}} : y^{\frac{2}{3}} = y^{\frac{2}{15}} = \sqrt[15]{y^2}$
   h) $z^{\frac{3}{4}} : z^{\frac{2}{3}} = z^{\frac{1}{12}} = \sqrt[12]{z}$
4. a) $2^{\frac{1}{2}} : 2^{\frac{1}{3}} = 2^{\frac{1}{6}} = \sqrt[6]{2}$
   b) $3^{\frac{1}{3}} : 3^{\frac{1}{4}} = 3^{\frac{1}{12}} = \sqrt[12]{3}$
   c) $5^{\frac{1}{2}} : 5^{\frac{1}{4}} = 5^{\frac{1}{4}} = \sqrt[4]{5}$
   d) $6^{\frac{1}{3}} : 6^{\frac{1}{5}} = 6^{\frac{2}{15}} = \sqrt[15]{6^2}$
   e) $5^{\frac{2}{3}} : 5^{\frac{1}{2}} = 5^{\frac{1}{6}} = \sqrt[6]{5}$
   f) $2^{\frac{3}{4}} : 2^{\frac{1}{2}} = 2^{\frac{1}{2}} = \sqrt{2}$
   g) $3^{\frac{2}{3}} : 3^{\frac{1}{4}} = 3^{\frac{5}{12}} = \sqrt[12]{3^5}$
   h) $4^{\frac{2}{5}} : 4^{\frac{1}{3}} = 4^{\frac{1}{15}} = \sqrt[15]{4}$
5. a) $a^{\frac{1}{3}} \cdot b^{\frac{1}{3}} = (ab)^{\frac{1}{3}} = \sqrt[3]{ab}$
   b) $x^{\frac{1}{4}} \cdot y^{\frac{1}{4}} = (xy)^{\frac{1}{4}} = \sqrt[4]{xy}$
   c) $c^{\frac{1}{2}} \cdot d^{\frac{1}{2}} = (cd)^{\frac{1}{2}} = \sqrt{cd}$
   d) $y^{\frac{1}{5}} \cdot z^{\frac{1}{5}} = (yz)^{\frac{1}{5}} = \sqrt[5]{yz}$
   e) $3^{\frac{1}{2}} \cdot 4^{\frac{1}{2}} = (3 \cdot 4)^{\frac{1}{2}} = \sqrt{12}$
   f) $5^{\frac{1}{3}} \cdot 6^{\frac{1}{3}} = (5 \cdot 6)^{\frac{1}{3}} = \sqrt[3]{30}$
   g) $8^{\frac{1}{4}} \cdot 2^{\frac{1}{4}} = (8 \cdot 2)^{\frac{1}{4}} = \sqrt[4]{16} = 2$
   h) $1^{\frac{1}{5}} \cdot 16^{\frac{1}{5}} = (1 \cdot 16)^{\frac{1}{5}} = \sqrt[5]{16}$
6. a) $a^{\frac{1}{2}} : b^{\frac{1}{2}} = \left(\frac{a}{b}\right)^{\frac{1}{2}} = \sqrt{\frac{a}{b}}$
   b) $x^{\frac{1}{4}} : y^{\frac{1}{4}} = \left(\frac{x}{y}\right)^{\frac{1}{4}} = \sqrt[4]{\frac{x}{y}}$
   c) $c^{\frac{1}{3}} : d^{\frac{1}{3}} = \left(\frac{c}{d}\right)^{\frac{1}{3}} = \sqrt[3]{\frac{c}{d}}$
   d) $x^{\frac{1}{5}} : y^{\frac{1}{5}} = \left(\frac{x}{y}\right)^{\frac{1}{5}} = \sqrt[5]{\frac{x}{y}}$
   e) $2^{\frac{1}{2}} : 3^{\frac{1}{2}} = \left(\frac{2}{3}\right)^{\frac{1}{2}} = \sqrt{\frac{2}{3}}$
   f) $1^{\frac{1}{5}} : 5^{\frac{1}{5}} = \left(\frac{1}{5}\right)^{\frac{1}{5}} = \sqrt[5]{\frac{1}{5}}$
   g) $32^{\frac{1}{4}} : 2^{\frac{1}{4}} = \left(\frac{32}{2}\right)^{\frac{1}{4}} = \sqrt[4]{16} = 2$
   h) $9^{\frac{1}{3}} : 3^{\frac{1}{3}} = \left(\frac{9}{3}\right)^{\frac{1}{3}} = \sqrt[3]{3}$
7. a) $(a^{\frac{1}{3}})^{\frac{1}{2}} = a^{\frac{1}{6}} = \sqrt[6]{a}$
   b) $(b^{\frac{1}{2}})^{\frac{1}{3}} = b^{\frac{1}{6}} = \sqrt[6]{b}$
   c) $(x^{\frac{1}{2}})^{\frac{1}{2}} = x^{\frac{1}{4}} = \sqrt[4]{x}$
   d) $(y^{\frac{1}{4}})^{\frac{1}{3}} = y^{\frac{1}{12}} = \sqrt[12]{y}$
   e) $(16^{\frac{1}{2}})^{\frac{1}{2}} = 16^{\frac{1}{4}} = \sqrt[4]{16} = 2$
   f) $(1^{\frac{1}{3}})^{\frac{1}{2}} = 1^{\frac{1}{6}} = \sqrt[6]{1} = 1$
   g) $(50^{\frac{1}{2}})^{\frac{1}{3}} = 50^{\frac{1}{6}} = \sqrt[6]{50}$
   h) $(25^{\frac{1}{3}})^{\frac{1}{4}} = 25^{\frac{1}{12}} = \sqrt[12]{25}$

13.3 Rechnen mit Wurzeln

1. a) $\sqrt[3]{64} = 4$ b) $\sqrt[4]{81} = 3$ c) $\sqrt[5]{32} = 2$ d) $\sqrt[6]{1} = 1$
2. a) $\sqrt[3]{125} = 5$ b) $\sqrt[4]{16} = 2$ c) $\sqrt[6]{64} = 2$
3. a) $\sqrt[3]{a^6} = a^2$ b) $\sqrt[4]{b^4} = b$ c) $\sqrt[5]{x^{10}} = x^2$

4. a) $\sqrt[3]{27a^6} = 3a^2$ b) $\sqrt[4]{10\,000x^4} = 10x$ c) $\sqrt[5]{32y^5} = 2y$

5. a) $\sqrt[3]{27} = 3$ b) $\sqrt[4]{10\,000} = 10$ c) $\sqrt[5]{32} = 2$ d) $\sqrt[6]{1} = 1$

6. a) $\sqrt[5]{a^5} = a$ b) $\sqrt[6]{b^6} = b$ c) $\sqrt[3]{27x^6} = 3x^2$ d) $\sqrt[4]{16y^4} = 2y$

7. a) $\sqrt[4]{\frac{1}{16}} = \frac{1}{2}$ b) $\sqrt[5]{b^5} = b$ c) $\sqrt[6]{\frac{64}{x^6}} = \frac{2}{x}$ d) $\sqrt[3]{\frac{8y^3}{27}} = \frac{2y}{3}$

8. a) $\sqrt{2^4} = \sqrt{16} = 4$ b) $\sqrt[3]{2^6} = \sqrt[3]{64} = 4$ c) $\sqrt[4]{4^2} = \sqrt[4]{16} = 2$
d) 7 e) 9

9. a) $\sqrt[5]{a^4}$ b) b c) $\sqrt[3]{x^5} = x\sqrt[3]{x^2}$ d) y^2 e) $\sqrt[5]{z^6} = z\sqrt[5]{z}$

10. a) $3^2 = 9$ b) $4^2 = 16$ c) $2^3 = 8$ d) $3^3 = 27$ e) $4^3 = 64$

11. a) $\sqrt{4} = 2$ b) $\sqrt{9} = 3$ c) $\sqrt{16} = 4$ d) $\sqrt[3]{8} = 2$ e) $\sqrt[4]{16} = 2$

12. a) a^2b^3 b) x^3y^2 c) $\sqrt{a}$ d) $\sqrt[3]{b}$ e) $\sqrt[4]{xy^2}$

13. a) 3 b) 2 c) 10 d) 2 e) $\frac{1}{2}$

14. a) a b) b c) c d) x^2 e) y^2

15. a) $\sqrt[6]{a^4} = \sqrt[3]{a^2}$ b) $\sqrt[9]{b^6} = \sqrt[3]{b^2}$ c) $\sqrt[12]{c^{10}} = \sqrt[6]{c^5}$
d) $\sqrt[6]{x^8} = \sqrt[3]{x^4} = x\sqrt[3]{x}$ e) $\sqrt[4]{y^6} = \sqrt[2]{y^3} = y\sqrt{y}$

16. a) 11 b) 15 c) 23 d) 31