Vertiefung zu Wurzeln und Potenzen

Beispiele

1. $\sqrt{\sqrt[3]{x^2}} = \sqrt{x^{\frac{2}{3}}} = x^{\frac{2}{3\cdot 2}} = x^{\frac{1}{3}} = \underline{\underline{\sqrt[3]{x}}}$

2. $\sqrt[3]{x^3 \cdot \sqrt{x^3}} = \sqrt[3]{x^3 \cdot x^{\frac{3}{2}}} = \sqrt[3]{x^{\frac{6+3}{2}}} = x^{\frac{9}{2\cdot 3}} = x^{\frac{3}{2}} = \underline{\underline{\sqrt{x^3}}}$

3. $\sqrt[5]{28} = 28^{\frac{1}{5}} = \underline{\underline{1{,}9473}}$

4. $\sqrt[3]{5{,}2^{2{,}5}} = 5{,}2^{\frac{2{,}5}{3}} = \underline{\underline{3{,}9507}}$

5. $2 \cdot \sqrt{3} + \sqrt{a} - \sqrt{3} - 5 \cdot \sqrt[3]{a} = \underline{\underline{\sqrt{3} + \sqrt{a} - 5 \cdot \sqrt[3]{a}}}$

6. $\sqrt[2]{a^6} = a^{\frac{6}{2}} = \underline{\underline{a^3}}$

7. $\sqrt[4]{25} = \sqrt[4]{5^2} = 5^{\frac{2}{4}} = 5^{\frac{1}{2}} = \underline{\underline{\sqrt{5}}}$

8. $\sqrt[3]{a} \cdot \sqrt[2]{a} = a^{\frac{1}{3}} \cdot a^{\frac{1}{2}} = a^{\frac{1}{3}+\frac{1}{2}} = a^{\frac{5}{6}} = \underline{\underline{\sqrt[6]{a^5}}}$

9. $\sqrt[3]{x^2} \cdot \sqrt[3]{x} = x^{\frac{2}{3}} \cdot x^{\frac{1}{3}} = x^{\frac{3}{3}} = \underline{\underline{x}}$

10. $\sqrt[3]{\sqrt[2]{2}} = \sqrt[3]{2^{\frac{1}{2}}} = 2^{\frac{1}{2\cdot 3}} = 2^{\frac{1}{6}} = \underline{\underline{\sqrt[6]{2}}}$

11. $\sqrt[4]{36} = \sqrt[4]{6^2} = 6^{\frac{2}{4}} = 6^{\frac{1}{2}} = \underline{\underline{\sqrt{6}}}$

12. $\sqrt[4]{16} = \sqrt[4]{4^2} = 4^{\frac{2}{4}} = 4^{\frac{1}{2}} = \sqrt{4} = \underline{\underline{2}}$ oder $\sqrt[4]{16} = \sqrt[4]{2^4} = 2^{\frac{4}{4}} = 2^1 = \underline{\underline{2}}$

13. $B = \frac{\sqrt{12} \cdot \sqrt{27}}{\sqrt{76{,}5^2 - 67{,}5^2}} = \frac{\sqrt{3 \cdot 4} \cdot \sqrt{3 \cdot 9}}{\sqrt{(76{,}5 - 67{,}5) \cdot (76{,}5 + 67{,}5)}} = \frac{2 \cdot \sqrt{3} \cdot 3 \cdot \sqrt{3}}{\sqrt{9 \cdot 144}} = \frac{6 \cdot 3}{3 \cdot 12} = \frac{1}{\underline{\underline{2}}}$

<u>Übungen</u>

1. $\sqrt[3m]{(a+b)^{2m}} \cdot \sqrt[3]{(a+b)} = (a+b)^{\frac{2m}{3m}} \cdot (a+b)^{\frac{1}{3}} = (a+b)^{\frac{2}{3}} \cdot (a+b)^{\frac{1}{3}} = (a+b)^{\frac{3}{3}} = \underline{\underline{a+b}}$

2. $\sqrt[5]{\sqrt[2]{x^5}} + \sqrt[4]{\sqrt[3]{x^6}} = \sqrt[5]{x^{\frac{5}{2}}} + \sqrt[4]{x^{\frac{6}{3}}} = x^{\frac{5}{2\cdot 5}} + x^{\frac{6}{3\cdot 4}} = x^{\frac{1}{2}} + x^{\frac{1}{2}} = 2\cdot x^{\frac{1}{2}} = \underline{\underline{2\sqrt{x}}}$

3. $\frac{\sqrt{a^4-a^2b^2}}{\sqrt[4]{a^4-b^4}} \cdot \frac{\sqrt[4]{a^4+2a^2b^2+b^4}}{a} = \frac{\sqrt{a^2\cdot(a^2-b^2)}}{\sqrt[4]{(a^2+b^2)\cdot(a^2-b^2)}} \cdot \frac{\sqrt[4]{(a^2+b^2)^2}}{a} = \frac{a\cdot(a^2-b^2)^{\frac{1}{2}}}{(a^2+b^2)^{\frac{1}{4}}\cdot(a^2-b^2)^{\frac{1}{4}}} \cdot \frac{(a^2+b^2)^{\frac{2}{4}}}{a} =$

$= (a^2-b^2)^{\frac{1}{2}-\frac{1}{4}}\cdot(a^2+b^2)^{\frac{1}{2}-\frac{1}{4}} = (a^2-b^2)^{\frac{1}{4}}\cdot(a^2+b^2)^{\frac{1}{4}} = \sqrt[4]{(a^2-b^2)\cdot(a^2+b^2)} = \underline{\underline{\sqrt[4]{(a^4-b^4)}}}$

4. $\sqrt[9]{\frac{(x+y)^6}{z^5}} \cdot \sqrt[9]{\frac{(x+y)^3}{z^4}} = \frac{(x+y)^{\frac{6}{9}}}{z^{\frac{5}{9}}} \cdot \frac{(x+y)^{\frac{3}{9}}}{z^{\frac{4}{9}}} = \frac{(x+y)^{\frac{9}{9}}}{z^{\frac{9}{9}}} = \underline{\underline{\frac{(x+y)}{z}}}$

5. $\sqrt[3]{\sqrt{x^2}} = \sqrt[3]{x} = \underline{\underline{x^{\frac{1}{3}}}}$

6. $\sqrt[3]{x^2\sqrt{x^2}} = \sqrt[3]{x^2\cdot x} = \sqrt[3]{x^3} = \underline{\underline{x}}$

7. $\left(\sqrt[3]{x}\right)^2\cdot\left(\sqrt[6]{x}\right)^3 = \left(x^{\frac{1}{3}}\right)^2\cdot\left(x^{\frac{1}{6}}\right)^3 = x^{\frac{2}{3}}\cdot x^{\frac{1}{2}} = x^{\frac{7}{6}} = \underline{\underline{\sqrt[6]{x^7}}} = x^{\frac{6}{6}}\cdot x^{\frac{1}{6}} = \underline{\underline{x\cdot\sqrt[6]{x}}}$

8. $\sqrt{\left(\frac{n^4\cdot x^3}{nx}\right)^2} = \frac{n^4\cdot x^3}{nx} = \underline{\underline{n^3\cdot x^2}}$

Mit Taschenrechner:

9. $\sqrt[5]{500} = \underline{\underline{3,4657}}$

10. $\sqrt[2,5]{3^{\frac{2}{3}}} = 3^{\frac{2}{3\cdot 2,5}} = 3^{\frac{2}{7,5}} = \underline{\underline{1,3404}}$

11. $\sqrt[5]{7} = \underline{\underline{1,4758}}$

12. $\sqrt[3,256789]{1^{2,56783219}} = \underline{\underline{1}}$

13. Schreiben Sie die Potenzen als Wurzelterm. In der Lösung sollen keine negativen Exponenten vorkommen.

a) $u^{-\frac{3}{4}} = \frac{1}{u^{\frac{3}{4}}} = \frac{1}{\sqrt[4]{u^3}} = \underline{\underline{\sqrt[4]{\frac{1}{u^3}}}}$

b) $5x^{-0,4} = \frac{5}{x^{\frac{2}{5}}} = \frac{5}{\sqrt[5]{x^2}} = \underline{\underline{5 \cdot \sqrt[5]{\frac{1}{x^2}}}}$

c) $x^{-\frac{1}{2}} + y^{-\frac{1}{2}} = \frac{1}{x^{\frac{1}{2}}} + \frac{1}{y^{\frac{1}{2}}} = \frac{1}{\sqrt{x}} + \frac{1}{\sqrt{y}} = \underline{\underline{\sqrt{\frac{1}{x}} + \sqrt{\frac{1}{y}}}}$

d) $\left(\frac{1}{n}\right)^{-\frac{5}{6}} = \frac{1}{\left(\frac{1}{n}\right)^{\frac{5}{6}}} = \frac{1}{\frac{1^{\frac{5}{6}}}{n^{\frac{5}{6}}}} = \frac{1}{\frac{1}{n^{\frac{5}{6}}}} = n^{\frac{5}{6}} = \underline{\underline{\sqrt[6]{n^5}}}$

e) $\frac{1}{a^{-\frac{2}{3}}} = \frac{1}{\frac{1}{a^{\frac{2}{3}}}} = a^{\frac{2}{3}} = \underline{\underline{\sqrt[3]{a^2}}}$

14. Schreiben Sie als Potenz.

a) $\sqrt[3]{a^4} = \underline{\underline{a^{\frac{4}{3}}}}$

b) $\sqrt[6]{\frac{u}{v}} = \underline{\underline{\left(\frac{u}{v}\right)^{\frac{1}{6}}}}$

c) $\sqrt[3]{a+b} = \underline{\underline{(a+b)^{\frac{1}{3}}}}$

d) $\sqrt[3]{x^3+y^3} = \underline{\underline{(x^3+y^3)^{\frac{1}{3}}}}$

e) $\sqrt[a]{x^{a-1}} = \underline{\underline{x^{\frac{a-1}{a}}}}$

15. Vereinfachen Sie.

a) $\sqrt[4]{a^2} = a^{\frac{2}{4}} = a^{\frac{1}{2}} = \underline{\underline{\sqrt{a}}}$

b) $\left(\sqrt[12]{x}\right)^4 = \left(x^{\frac{1}{12}}\right)^4 = x^{\frac{4}{12}} = x^{\frac{1}{3}} = \underline{\underline{\sqrt[3]{x}}}$

c) $\sqrt[4]{25x^2y^6} = \left(5^2x^2y^6\right)^{\frac{1}{4}} = 5^{\frac{2}{4}}x^{\frac{2}{4}}y^{\frac{6}{4}} = 5^{\frac{1}{2}}x^{\frac{1}{2}}y^{\frac{3}{2}} = 5^{\frac{1}{2}}x^{\frac{1}{2}}y^{\frac{1}{2}}y^{\frac{1}{2}}y^{\frac{1}{2}} = \left(5xy^3\right)^{\frac{1}{2}} = \underline{\underline{\sqrt{5xy^3}}}$

d) $\left(\sqrt[2n]{a}\right)^{3n} = \left(a^{\frac{1}{2n}}\right)^{3n} = a^{\frac{3n}{2n}} = a^{\frac{3}{2}} = \underline{\underline{\sqrt{a^3}}}$

e) $\sqrt[5n]{\frac{a^{10}b^5}{c^{20}}} = \left(\frac{a^{10}b^5}{c^{20}}\right)^{\frac{1}{5n}} = \frac{\left(a^{10}b^5\right)^{\frac{1}{5n}}}{\left(c^{20}\right)^{\frac{1}{5n}}} = \frac{a^{\frac{10}{5n}}b^{\frac{5}{5n}}}{c^{\frac{20}{5n}}} = \frac{a^{\frac{2}{n}}b^{\frac{1}{n}}}{c^{\frac{4}{n}}} = \left(\frac{a^2b}{c^4}\right)^{\frac{1}{n}} = \underline{\underline{\sqrt[n]{\frac{a^2b}{c^4}}}}$

f) $\sqrt[3]{2}\cdot\sqrt[3]{4} = 2^{\frac{1}{3}}\cdot 4^{\frac{1}{3}} = 2^{\frac{1}{3}}\cdot\left(2^2\right)^{\frac{1}{3}} = 2^{\frac{1}{3}}\cdot 2^{\frac{2}{3}} = 2^{\frac{1}{3}+\frac{2}{3}} = 2^{\frac{3}{3}} = \underline{\underline{2}}$

g) $\sqrt[5]{\sqrt{3}-1}\cdot\sqrt[5]{\sqrt{3}+1} = \left(\sqrt{3}-1\right)^{\frac{1}{5}}\cdot\left(\sqrt{3}+1\right)^{\frac{1}{5}} = \left[\left(\sqrt{3}-1\right)\cdot\left(\sqrt{3}+1\right)\right]^{\frac{1}{5}} = (3-1)^{\frac{1}{5}} = \underline{\underline{\sqrt[5]{2}}}$

h) $\sqrt[5]{a^2} : \sqrt[5]{a} = \frac{a^{\frac{2}{5}}}{a^{\frac{1}{5}}} = a^{\frac{2}{5}-\frac{1}{5}} = a^{\frac{1}{5}} = \underline{\underline{\sqrt[5]{a}}}$

i) $\sqrt[n]{a^{n-2}}\cdot\sqrt[n]{a^{n+2}} = a^{\frac{n-2}{n}}\cdot a^{\frac{n+2}{n}} = a^{\frac{n-2}{n}+\frac{n+2}{n}} = a^{\frac{n-2+n+2}{n}} = a^{\frac{2n}{n}} = \underline{\underline{a^2}}$

j) $\sqrt[10]{15a^5b^7} : \sqrt[10]{3ab} = \frac{\left(15a^5b^7\right)^{\frac{1}{10}}}{(3ab)^{\frac{1}{10}}} = \left(\frac{15a^5b^7}{3ab}\right)^{\frac{1}{10}} = \left(5a^4b^6\right)^{\frac{1}{10}} = \underline{\underline{\sqrt[10]{5a^4b^6}}}$

16. Schreiben Sie mit einem einzigen Wurzelzeichen. Der Wurzelexponent soll so klein wie möglich sein.

a) $\sqrt{3}\cdot\sqrt[4]{3} = 3^{\frac{1}{2}}\cdot 3^{\frac{1}{4}} = 3^{\frac{1}{2}+\frac{1}{4}} = 3^{\frac{3}{4}} = \underline{\underline{\sqrt[4]{3^3}}}$

b) $\sqrt[4]{a}\cdot\sqrt[6]{a^{-1}} = a^{\frac{1}{4}}\cdot a^{-\frac{1}{6}} = a^{\frac{1}{4}-\frac{1}{6}} = a^{\frac{3-2}{12}} = a^{\frac{1}{12}} = \underline{\underline{\sqrt[12]{a}}}$

c) $\sqrt[3]{t^2}\cdot\sqrt[4]{t^3}\cdot\sqrt[6]{t^5} = t^{\frac{2}{3}}\cdot t^{\frac{3}{4}}\cdot t^{\frac{5}{6}} = t^{\frac{2}{3}+\frac{3}{4}+\frac{5}{6}} = t^{\frac{8+9+10}{12}} = t^{\frac{27}{12}} = t^{\frac{24}{12}}\cdot t^{\frac{3}{12}} = t^2\cdot t^{\frac{1}{4}} = \underline{\underline{t^2\cdot\sqrt[4]{t}}}$ oder $t^{\frac{27}{12}} = t^{\frac{9}{4}} = \underline{\underline{\sqrt[4]{t^9}}}$

d) $\sqrt[6]{m^2-n^2} : \sqrt[3]{m-n} = \frac{\left(m^2-n^2\right)^{\frac{1}{6}}}{(m-n)^{\frac{1}{3}}} = \frac{\left[(m-n)\cdot(m+n)\right]^{\frac{1}{6}}}{(m-n)^{\frac{1}{3}}} = \frac{(m-n)^{\frac{1}{6}}\cdot(m+n)^{\frac{1}{6}}}{(m-n)^{\frac{1}{3}}} = (m-n)^{\frac{1}{6}-\frac{1}{3}}\cdot(m+n)^{\frac{1}{6}} =$

$(m-n)^{-\frac{1}{6}}\cdot(m+n)^{\frac{1}{6}} = \frac{(m+n)^{\frac{1}{6}}}{(m-n)^{\frac{1}{6}}} = \left(\frac{m+n}{m-n}\right)^{\frac{1}{6}} = \underline{\underline{\sqrt[6]{\frac{m+n}{m-n}}}}$

e) $\sqrt[8]{\frac{x}{y}}\cdot\sqrt[12]{\frac{y}{x}} = \left(\frac{x}{y}\right)^{\frac{1}{8}}\cdot\left(\frac{y}{x}\right)^{\frac{1}{12}} = \frac{x^{\frac{1}{8}}}{y^{\frac{1}{8}}}\cdot\frac{y^{\frac{1}{12}}}{x^{\frac{1}{12}}} = x^{\frac{1}{8}-\frac{1}{12}}\cdot y^{\frac{1}{12}-\frac{1}{8}} = x^{\frac{3-2}{24}}\cdot y^{\frac{2-3}{24}} = x^{\frac{1}{24}}\cdot y^{-\frac{1}{24}} = \left(\frac{x}{y}\right)^{\frac{1}{24}} = \underline{\underline{\sqrt[24]{\frac{x}{y}}}}$

f) $\sqrt[10]{\sqrt[10]{10}} = \sqrt[10]{10^{\frac{1}{10}}} = 10^{\frac{1}{10\cdot 10}} = 10^{\frac{1}{100}} = \underline{\underline{\sqrt[100]{10}}}$

g) $\sqrt{\sqrt[3]{a}} = \sqrt{a^{\frac{1}{3}}} = a^{\frac{1}{3\cdot 2}} = a^{\frac{1}{6}} = \underline{\underline{\sqrt[6]{a}}}$

h) $\sqrt[5]{\sqrt[3]{m^5n^{10}}} = \sqrt[5]{\left(m^5n^{10}\right)^{\frac{1}{3}}} = \left(m^5n^{10}\right)^{\frac{1}{3\cdot 5}} = \left(m^5n^{10}\right)^{\frac{1}{15}} = \left(m^5\right)^{\frac{1}{15}}\cdot\left(n^{10}\right)^{\frac{1}{15}} = m^{\frac{5}{15}}\cdot n^{\frac{10}{15}} =$

$m^{\frac{1}{3}}\cdot n^{\frac{2}{3}} = \left(m\cdot n^2\right)^{\frac{1}{3}} = \underline{\underline{\sqrt[3]{m\cdot n^2}}}$

17. Schreiben Sie mit einem einzigen Wurzelzeichen.

a) $\sqrt{2\sqrt[3]{4}} = \sqrt{2\sqrt[3]{2^2}} = \sqrt{2\cdot 2^{\frac{2}{3}}} = \sqrt{2^{1+\frac{2}{3}}} = \sqrt{2^{\frac{3+2}{3}}} = \left(2^{\frac{5}{3}}\right)^{\frac{1}{2}} = 2^{\frac{5}{6}} = \sqrt[6]{2^5}$

b) $\sqrt[3]{a\sqrt{a}} = \sqrt[3]{a\cdot a^{\frac{1}{2}}} = \sqrt[3]{a^{1+\frac{1}{2}}} = \sqrt[3]{a^{\frac{2+1}{2}}} = \sqrt[3]{a^{\frac{3}{2}}} = \left(a^{\frac{3}{2}}\right)^{\frac{1}{3}} = a^{\frac{3}{6}} = a^{\frac{1}{2}} = \sqrt{a}$

c) $\sqrt[4]{b^3\cdot\sqrt[3]{b}} = \sqrt[4]{b^3\cdot b^{\frac{1}{3}}} = \sqrt[4]{b^{3+\frac{1}{3}}} = \sqrt[4]{b^{\frac{9+1}{3}}} = \sqrt[4]{b^{\frac{10}{3}}} = \left(b^{\frac{10}{3}}\right)^{\frac{1}{4}} = b^{\frac{10}{12}} = b^{\frac{5}{6}} = \sqrt[6]{b^5}$

d) $\sqrt[3]{x^2\cdot y\cdot\sqrt{xy^{-1}}} = \sqrt[3]{x^2\cdot y\cdot\sqrt{x\cdot\frac{1}{y}}} = \sqrt[3]{x^2\cdot y\cdot\frac{x^{\frac{1}{2}}}{y^{\frac{1}{2}}}} = \sqrt[3]{x^{2+\frac{1}{2}}\cdot y^{1-\frac{1}{2}}} = \sqrt[3]{x^{\frac{4+1}{2}}\cdot y^{\frac{2-1}{2}}} = \sqrt[3]{x^{\frac{5}{2}}\cdot y^{\frac{1}{2}}} = \left(x^{\frac{5}{2}}\right)^{\frac{1}{3}}\cdot\left(y^{\frac{1}{2}}\right)^{\frac{1}{3}} =$

$x^{\frac{5}{6}}\cdot y^{\frac{1}{6}} = \left(x^5\cdot y\right)^{\frac{1}{6}} = \sqrt[6]{x^5\cdot y}$

e) $\sqrt[3]{a\sqrt{a\sqrt{a}}} = \sqrt[3]{a\sqrt{a\cdot a^{\frac{1}{2}}}} = \sqrt[3]{a\sqrt{a^{1+\frac{1}{2}}}} = \sqrt[3]{a\sqrt{a^{\frac{3}{2}}}} = \sqrt[3]{a\cdot a^{\frac{3}{2\cdot 2}}} = \sqrt[3]{a\cdot a^{\frac{3}{4}}} = \sqrt[3]{a^{1+\frac{3}{4}}} = \sqrt[3]{a^{\frac{7}{4}}} =$

$a^{\frac{7}{4\cdot 3}} = a^{\frac{7}{12}} = \sqrt[12]{a^7}$

18. Berechnen und vereinfachen Sie soweit als möglich:

$$E = \frac{\left(\sqrt[4]{u}\right)^3\cdot s^{-\frac{1}{3}}\cdot\sqrt{s^{-1}}}{\left(\frac{1}{\sqrt[12]{u}}\right)^5\cdot s^{-\frac{5}{6}}\cdot\left(\sqrt[3]{u}\right)^2} = \frac{\left(u^{\frac{1}{4}}\right)^3\cdot s^{-\frac{1}{3}}\cdot s^{-\frac{1}{2}}}{\left(\frac{1}{u^{\frac{1}{12}}}\right)^5\cdot s^{-\frac{5}{6}}\cdot\left(u^{\frac{1}{3}}\right)^2} = \frac{u^{\frac{3}{4}}\cdot s^{-\frac{1}{3}}\cdot s^{-\frac{1}{2}}}{\frac{1}{u^{\frac{5}{12}}}\cdot s^{-\frac{5}{6}}\cdot u^{\frac{2}{3}}} = \frac{u^{\frac{3}{4}}\cdot s^{-\frac{1}{3}}\cdot s^{-\frac{1}{2}}}{u^{-\frac{5}{12}}\cdot s^{-\frac{5}{6}}\cdot u^{\frac{2}{3}}} = u^{\frac{3}{4}+\frac{5}{12}-\frac{2}{3}}\cdot s^{-\frac{2}{6}-\frac{3}{6}+\frac{5}{6}} = u^{\frac{1}{2}} = \sqrt{u}$$

19. Vereinfachen Sie den Ausdruck E und bestimmen Sie nachher seinen Wert für a = 2/7 und b = 31,5 :

$$E = \frac{a\sqrt{b}+b\sqrt{a}}{\sqrt{a}+\sqrt{b}} = \frac{\sqrt{a}\cdot\sqrt{b}\cdot\left(\sqrt{a}+\sqrt{b}\right)}{\sqrt{a}+\sqrt{b}} = \sqrt{ab} = \sqrt{\frac{2}{7}\cdot\frac{63}{2}} = \sqrt{9} = 3$$

20. Berechnen und vereinfachen Sie soweit als möglich:

$$E = \frac{\sqrt[12]{v^5}\cdot t^{\frac{5}{6}}\cdot\sqrt{t^{-1}}}{v^{-\frac{3}{4}}\cdot t^{\frac{1}{3}}\cdot\sqrt[3]{v^2}} = \frac{v^{\frac{5}{12}}\cdot t^{\frac{5}{6}}\cdot t^{-\frac{1}{2}}}{v^{-\frac{3}{4}}\cdot t^{\frac{1}{3}}\cdot v^{\frac{2}{3}}} = v^{\frac{5+9-8}{12}}\cdot t^{\frac{5}{6}-\frac{3}{6}-\frac{2}{6}} = v^{\frac{1}{2}} = \sqrt{v}$$

21. Vereinfachen Sie den folgenden Ausdruck soweit als möglich:

$$5a\cdot\sqrt{a\sqrt{a\sqrt{a}}}-2\cdot\sqrt{a^3\cdot\sqrt[4]{a^3}}+a\cdot\sqrt[4]{a^5\cdot\frac{1}{\sqrt{a^3}}}=5a\cdot\sqrt{a\sqrt{a\cdot a^{\frac{1}{2}}}}-2\cdot\sqrt{a^3\cdot a^{\frac{3}{4}}}+a\cdot\sqrt[4]{a^5\cdot\frac{1}{a^{\frac{3}{2}}}}=$$

$$=5a\cdot\sqrt{a\sqrt{a^{\frac{3}{2}}}}-2\cdot\sqrt{a^{\frac{15}{4}}}+a\cdot\sqrt[4]{a^{\frac{7}{2}}}=5a\cdot\sqrt{a\cdot a^{\frac{3}{2\cdot 2}}}-2\cdot a^{\frac{15}{4\cdot 2}}+a\cdot a^{\frac{7}{2\cdot 4}}=5a\cdot\sqrt{a\cdot a^{\frac{3}{4}}}-2\cdot a^{\frac{15}{8}}+a\cdot a^{\frac{7}{8}}=$$

$$=5a\cdot\sqrt{a^{\frac{7}{4}}}-2\cdot a^{\frac{15}{8}}+a^{\frac{15}{8}}=5a\cdot a^{\frac{7}{4\cdot 2}}-2\cdot a^{\frac{15}{8}}+a^{\frac{15}{8}}=5a^{\frac{15}{8}}-2\cdot a^{\frac{15}{8}}+a^{\frac{15}{8}}=$$

$$=a^{\frac{15}{8}}\cdot(5-2+1)=4\cdot a^{\frac{15}{8}}=\underline{\underline{4\cdot\sqrt[8]{a^{15}}}}=4\cdot a^{\frac{8}{8}}\cdot a^{\frac{7}{8}}=\underline{\underline{4\cdot a\cdot\sqrt[8]{a^7}}}$$

22. Vereinfachen Sie den folgenden Ausdruck soweit als möglich:

$$a^4\cdot\left(1-\frac{2}{a}-\frac{2}{a^2}\right)^2-a^4\cdot\left(1-\frac{2}{a^2}\right)^2=a^4\cdot\left(\frac{a^2}{a^2}-\frac{2a}{a^2}-\frac{2}{a^2}\right)^2-a^4\cdot\left(\frac{a^2}{a^2}-\frac{2}{a^2}\right)^2=a^4\cdot\frac{(a^2-2a-2)^2}{a^4}-a^4\cdot\frac{(a^2-2)^2}{a^4}=$$

$$=(a^2-2a-2)^2-(a^2-2)^2=a^4-4a^3+8a+4-a^4+4a^2-4=-4a^3+8a+4a^2=$$

$$=-4a\cdot(a^2-2-a)=\underline{\underline{-4a\cdot(a-2)\cdot(a+1)}}$$

23. Vereinfachen Sie den folgenden Ausdruck soweit als möglich:

$$A=\left(1+\frac{2}{a}\right)^2\cdot\left\{\frac{1}{a}-\left(\frac{a}{2}-1\right)^{-1}\right\}^{-2}=\left(\frac{a+2}{a}\right)^2\cdot\left\{\frac{1}{a}-\left(\frac{a-2}{2}\right)^{-1}\right\}^{-2}=\left(\frac{a+2}{a}\right)^2\cdot\left\{\frac{1}{a}-\left(\frac{2}{a-2}\right)\right\}^{-2}=$$

$$=\left(\frac{a+2}{a}\right)^2\cdot\left\{\frac{a-2-2a}{a\cdot(a-2)}\right\}^{-2}=\frac{(a+2)^2}{a^2}\cdot\frac{\left[a\cdot(a-2)\right]^2}{(-a-2)^2}=\frac{(a+2)^2\cdot a^2\cdot(a-2)^2}{a^2\cdot\left[(-1)\cdot(a+2)\right]^2}=\frac{(a+2)^2\cdot a^2\cdot(a-2)^2}{a^2\cdot(a+2)^2}=\underline{\underline{(a-2)^2}}$$

<u>Übungen Potenzieren und Radizieren</u>

1. $(3ax^3)^7 = 3^7a^7x^{21} = \underline{\underline{2187a^7x^{21}}}$

2. $-a^4 \cdot (-a^6) = \underline{\underline{a^{10}}}$

3. $(2a^2b)^3 \cdot (3ab^2)^3 = (2a^2b \cdot 3ab^2)^3 = (6a^3b^3)^3 = \underline{\underline{216a^9b^9}}$

4. $100 \cdot (3 \cdot 10^{-2})^{-2} = 100 \cdot \left(3 \cdot \frac{1}{10^2}\right)^{-2} = 10^2 \cdot \frac{1}{3^2} \cdot 10^4 = \underline{\underline{\frac{10^6}{9}}}$

5. $-\left[3 \cdot \left(-\frac{3}{10}\right)^{-2}\right]^3 = -\left[3^3 \cdot \left(-\frac{3}{10}\right)^{-6}\right] = -\left[3^3 \cdot \left(-\frac{10}{3}\right)^6\right] = -\frac{3^3 \cdot 10^6}{3^6} = -\frac{10^6}{3^3} = -\underline{\underline{\frac{10^6}{27}}}$

6. $\sqrt[3]{27^4} = \sqrt[3]{(3^3)^4} = \sqrt[3]{3^{12}} = 3^{\frac{12}{3}} = \underline{\underline{3^4}} = \underline{\underline{81}}$

7. $\sqrt[x]{a^x} = a^{\frac{x}{x}} = \underline{\underline{a}}$

8. $(\sqrt[3]{x})^2 \cdot (\sqrt[6]{x})^3 = \left(x^{\frac{1}{3}}\right)^2 \cdot \left(x^{\frac{1}{6}}\right)^3 = x^{\frac{2}{3}} \cdot x^{\frac{3}{6}} = x^{\frac{7}{6}} = x^{\frac{6}{6}} \cdot x^{\frac{1}{6}} = \underline{\underline{x \cdot \sqrt[6]{x}}}$

9. $\frac{\sqrt{98}}{49 \cdot \sqrt{2}} = \frac{\sqrt{2 \cdot 7 \cdot 7}}{7 \cdot 7 \cdot \sqrt{2}} = \underline{\underline{\frac{1}{7}}}$

10. $\sqrt{\left(\frac{n^4x^3}{nx}\right)^2} = \frac{n^4x^3}{nx} = \underline{\underline{n^3x^2}}$

11. $(\sqrt[4]{49a})^2 \div 14 = \frac{\sqrt{49a}}{14} = \frac{7 \cdot \sqrt{a}}{14} = \underline{\underline{\frac{\sqrt{a}}{2}}}$

12. $\left[(n^2 \cdot x^3)^2\right]^{-2} = (n^2 \cdot x^3)^{-4} = \underline{\underline{n^{-8} \cdot x^{-12}}}$