

4 Zerlegen in Faktoren (Ausklammern)

4.11 Übungen Frommenwiler

29. a) $-(b-a)$

b) $-(2m+3n)$

c) $-(2a^2-3a+7)$

30. a) $4a(a^3-2a^2+5a-1)$

b) $3ab(3ab-4a+b)$

c) $4x^2y(-2y^2-8x+3)$

d) $(x-y)(n-1)$

e) $(3p-2q)[4a-5b-(a+5b)] = (3p-2q)(4a-5b-a-5b) = \underline{\underline{(3p-2q)(3a-10b)}}$

f) $m(a-b)-n(a-b) = \underline{\underline{(a-b)(m-n)}}$

g) $\underbrace{-r(2-a)}_{(-1) \text{ ausklammern}} - r^2(2-a) + r^3(2-a) = (2-a)(r^3-r^2-r) = \underline{\underline{r(2-a)(r^2-r-1)}}$

h) $(x+1)(x-y) - (x-3)(x-y) + (x+2)\underbrace{(-1)(x-y)}_{(-1) \text{ ausklammern}} =$
 $(x-y)[x+1-(x-3)+(-x-2)] = (x-y)[x+1-x+3-x-2] = \underline{\underline{(x-y)(2-x)}}$

31. a) $a(c-d) + b(c-d) = \underline{\underline{(a+b)(c-d)}}$

b) $a(b-c) - (b-c) = \underline{\underline{(a-1)(b-c)}}$

c) $x^2(2x-3) + 5(2x-3) = \underline{\underline{(x^2+5)(2x-3)}}$

d) $-2a^3(3a-5) + 5(3a-5) = \underline{\underline{(3a-5)(5-2a^3)}} = \underline{\underline{(5-3a)(2a^3-5)}}$

e) $5a(4b-1) + (4b-1) = \underline{\underline{(5a+1)(4b-1)}}$

f) $(a-1)(x-y) - (x-y) = (x-y)(a-1-1) = \underline{\underline{(x-y)(a-2)}}$

g) $4(4ac - 6bc - 10ad + 15bd) = 4[2c(2a-3b) - 5d(2a-3b)] = \underline{\underline{4(2a-3b)(2c-5d)}}$

h) $2(5abc - 2 - a + 10bc) = 2[5bc(a+2) - (a+2)] = \underline{\underline{2(a+2)(5bc-1)}}$

i) $5y^2(5xy - 10yz - 3x^2 + 6xz) = 5y^2[5y(x-2z) - 3x(x-2z)] = \underline{\underline{5y^2(x-2z)(5y-3x)}}$

32. a) $x(a-b+2) - y(a-b+2) = \underline{\underline{(x-y)(a-b+2)}}$

b) $a(2x-5y+1) - b(2x-5y+1) = \underline{\underline{(a-b)(2x-5y+1)}}$

c) $2x^2 + 5x - 12xz - 30z + 10xy + 25y =$
 $x(2x+5) - 6z(2x+5) + 5y(2x+5) = \underline{\underline{(2x+5)(x+5y-6z)}}$

d) $10ac - 5bc + 4ae - 2be - 2ad + bd =$
 $5c(2a-b) + 2e(2a-b) - d(2a-b) = \underline{\underline{(2a-b)(5c-d+2e)}}$

oder (Terme zuerst ordnen)

$$\square 10ac + 4ae - 2ad - 5bc - 2be + bd =$$

$$2a(5c + 2e - d) - b(5c + 2e - d) = \underline{\underline{(5c + 2e - d)(2a - b)}}$$

$$e) 5(6r^2 + 6s - 9rs - 4t - 4r + 6rt) = 5(6r^2 - 4r - 9rs + 6s + 6rt - 4t) =$$

$$5[2r(3r - 2) - 3s(3r - 2) + 2t(3r - 2)] = \underline{\underline{5(3r - 2)(2r - 3s + 2t)}}$$

$$f) 2a^2(19a - 18c) - 3a(19a - 18c) - (19a - 18c) = \underline{\underline{(19a - 18c)(2a^2 - 3a - 1)}}$$

$$33. a) \underline{\underline{(1-x)(1+x)}}$$

$$b) \underline{\underline{(3a-7b)(3a+7b)}}$$

$$c) u(u^2 - 25) = \underline{\underline{u(u-5)(u+5)}}$$

$$d) 2(4m^2 - 49n^2) = \underline{\underline{2(2m-7n)(2m+7n)}}$$

$$e) 2xy(x^2 - 9y^2) = \underline{\underline{2xy(x-3y)(x+3y)}}$$

$$f) \underbrace{(h^2 - 4)}_{(h-2)(h+2)}(h^2 + 4) = \underline{\underline{(h-2)(h+2)(h^2 + 4)}}$$

$$g) \underbrace{25x^2}_{\Delta^2} - \underbrace{(2x+3)^2}_{\square^2} = \left[\underbrace{5x}_{\Delta} - \underbrace{(2x+3)}_{\square} \right] \left[\underbrace{5x}_{\Delta} + \underbrace{(2x+3)}_{\square} \right] =$$

$$(3x-3)(7x+3) = \underline{\underline{3(x-1)(7x+3)}}$$

$$h) \underbrace{(a^2-2)^2}_{\Delta^2} - \underbrace{(a+1)^2}_{\square^2} = \left[\underbrace{(a^2-2)}_{\Delta} - \underbrace{(a+1)}_{\square} \right] \left[\underbrace{(a^2-2)}_{\Delta} + \underbrace{(a+1)}_{\square} \right] = \underline{\underline{(a^2-a-3)(a^2+a-1)}}$$

$$i) \underbrace{4(a-b)^2}_{\Delta^2} - \underbrace{81c^2}_{\sigma^2} = [2(a-b) - 9c][2(a-b) + 9c] = \underline{\underline{(2a-2b-9c)(2a-2b+9c)}}$$

$$j) \underbrace{100t^2}_{\Delta^2} - \underbrace{9(u-t)^2}_{\sigma^2} = [10t - 3(u-t)][10t + 3(u-t)] = \underline{\underline{(13t-3u)(7t+3u)}}$$

$$k) (2d-1)^2 - 36(2-d)^2 = [2d-1-6(2-d)][2d-1+6(2-d)] = \underline{\underline{(8d-13)(11-4d)}}$$

$$l) a^4 - (a^2 + a + 1)^2 = [a^2 - (a^2 + a + 1)][a^2 + (a^2 + a + 1)] = \\ (-a-1)(2a^2 + a + 1) = \underline{\underline{-(a+1)(2a^2 + a + 1)}}$$

$$34. a) \underbrace{a^2 + 2ae + e^2}_{(a+e)^2} - b^2 = \underbrace{(a+e)^2}_{\Delta^2} - \underbrace{b^2}_{\sigma^2} = \underline{\underline{(a+e-b)(a+e+b)}}$$

$$b) \underbrace{g^2 - 2gs + s^2}_{(g-s)^2} - f^2 = \underbrace{(g-s)^2}_{\Delta^2} - \underbrace{f^2}_{\sigma^2} = \underline{\underline{(g-s-f)(g-s+f)}}$$

$$c) \underbrace{x^2 - 6x + 3^2}_{(x-3)^2} - y^2 = \underbrace{(x-3)^2}_{\Delta^2} - \underbrace{y^2}_{\sigma^2} = \underline{\underline{(x-3-y)(x-3+y)}}$$

$$d) x^2 - \underbrace{(y^2 - 2yz + z^2)}_{(y-z)^2} = \underbrace{x^2}_{\Delta^2} - \underbrace{(y-z)^2}_{\sigma^2} = [x - (y-z)][x + (y-z)] = \underline{\underline{(x-y+z)(x+y-z)}}$$

$$e) 4a^2 - \underbrace{(9x^2 - 6xy + y^2)}_{(3x-y)^2} = \underbrace{4a^2}_{\Delta^2} - \underbrace{(3x-y)^2}_{\sigma^2} = \\ [2a - (3x-y)][2a + (3x-y)] = \underline{\underline{(2a-3x+y)(2a+3x-y)}}$$

$$f) 1 - \underbrace{(u^2 + 2ux + x^2)}_{(u+x)^2} = \underbrace{1}_{\Delta^2} - \underbrace{(u+x)^2}_{\sigma^2} = [1 - (u+x)][1 + (u+x)] = \underline{\underline{(1-u-x)(1+u+x)}}$$

$$g) \underbrace{9x^2 - 6xy + y^2}_{(3x-y)^2} - z^2 = \underbrace{(3x-y)^2}_{\Delta^2} - \underbrace{z^2}_{\sigma^2} = \underline{\underline{(3x-y-z)(3x-y+z)}}$$

$$h) 4a^2 - \underbrace{(25b^2 - 30bc + 9c^2)}_{(5b-3c)^2} = \underbrace{4a^2}_{\Delta^2} - \underbrace{(5b-3c)^2}_{\sigma^2} = \\ [2a - (5b-3c)][2a + (5b-3c)] = \underline{\underline{(2a-5b+3c)(2a+5b-3c)}}$$

$$i) \underbrace{(49u^2 - 28uv + 4v^2)}_{(7u-2v)^2} - 25t^2 = \underbrace{(7u-2v)^2}_{\Delta^2} - \underbrace{25t^2}_{\sigma^2} = \underline{\underline{(7u-2v-5t)(7u-2v+5t)}}$$

$$35. a) x^2 + 8x + 15 = \underline{\underline{(x+3)(x+5)}} \\ \quad \quad \quad (+3)(+5) \quad (+3)(+5)$$

$$b) a^2 - 1a - 12 = \underline{\underline{(a+3)(a-4)}} \\ \quad \quad \quad (+3)(-4) \quad (+3)(-4)$$

$$c) r^2 - 15r + 54 = \underline{\underline{(r-6)(r-9)}} \\ \quad \quad \quad (-6)(-4) \quad (-6)(-9)$$

$$d) b^2 + 3b - 28 = \underline{\underline{(b+7)(b-4)}} \\ \quad \quad \quad (+7)(-4) \quad (+7)(-4)$$

$$e) h^2 + 24h + 135 = \underline{\underline{(h+9)(h+15)}} \\ \quad \quad \quad (+9)(+15) \quad (+9)(+15)$$

$$f) u^2 + 2u - 143 = \underline{\underline{(u+13)(u-11)}} \\ \quad \quad \quad (+13)(-11) \quad (+13)(-11)$$

$$g) v^2 - 12v + 36 = (v-6)(v-6) = \underline{\underline{(v-6)^2}} \\ \quad \quad \quad (-6)(-6) \quad (-6)(-6)$$

$$h) m^2 - 43m - 240 = \underline{\underline{(m+5)(m-48)}} \\ \quad \quad \quad (+5)(-48) \quad (+5)(-48)$$

$$i) t^2 + 2t - 168 = \underline{\underline{(t+14)(t-12)}} \\ \quad \quad \quad (+14)(-12) \quad (+14)(-12)$$

$$j) \quad a^4 + 20a^2 + 96 = \underbrace{(a^2 + 8)}_{(+8)(+12)} \underbrace{(a^2 + 12)}_{(+8)(+12)}$$

$$k) \quad b^4 - 4b^2 - 77 = \underbrace{(b^2 + 7)}_{(+7)(+11)} \underbrace{(b^2 - 11)}_{(+7)(-11)}$$

$$l) \quad x^4 + 3x^2 - 108 = \underbrace{(x^2 + 12)}_{(+12)(+9)} \underbrace{(x^2 - 9)}_{(+12)(-9)} = \underbrace{(x^2 + 12)}_{(+12)(+9)} \underbrace{(x + 3)}_{(+12)(+9)} \underbrace{(x - 3)}_{(+12)(-9)}$$

$$36. \quad a) \quad 3a^2 + 13a - 30 = \underbrace{(3a - 5)}_{3a-1a} \underbrace{(a + 6)}_{(-5)(+6)}$$

$$b) \quad \overbrace{9m^2 + 48m + 64}^{\text{Binom}} = \underbrace{(3m + 8)^2}_{3m-3m \quad (+8)(+8)}$$

$$c) \quad 6x^2 - 29x + 20 = \underbrace{(6x - 5)}_{6x-x} \underbrace{(x - 4)}_{(-5)(-4)}$$

$$d) \quad 35y^2 + 3y - 2 = \underbrace{(7y + 2)}_{7y-5y} \underbrace{(5y - 1)}_{(+2)(-1)}$$

$$e) \quad -4 + 4k - k^2 = \underbrace{(2 - k)}_{(+2)(-2)} \underbrace{(-2 + k)}_{(-k)(+k)} = \underbrace{(2 - k)(k - 2)}_{(+2)(-2) \quad (-k)(+k)}$$

$$f) \quad -28 + 15b - 2b^2 = \underbrace{(-7 + 2b)}_{(-7)(+4)} \underbrace{(4 - b)}_{(2b)(-b)} = \underbrace{(2b - 7)}_{(-7)(+4)} \underbrace{(4 - b)}_{(2b)(-b)} = \underbrace{(7 - 2b)(b - 4)}_{\text{Frommenwiler}}$$

$$37. \quad a) \quad a^2 + 28ab + 75b^2 = \underbrace{(a + 3b)}_{(+3)(+25)} \underbrace{(a + 25b)}_{(+3)(+25)}$$

$$b) \quad x^2 - 15xy + 54y^2 = \underbrace{(x - 6y)}_{(-6)(-9)} \underbrace{(x - 9y)}_{(-6)(-9)}$$

$$c) \quad p^2 - 8pq - 65q^2 = \underbrace{(p + 5q)}_{(+5)(-13)} \underbrace{(p - 13q)}_{(+5)(-13)}$$

$$d) 2 \left(\begin{array}{cc} a^2 & +11 ab & +24 b^2 \\ \text{(+3)(+8)} & \text{(+3)(+8)} & \end{array} \right) = \underline{\underline{2(a+3b)(a+8b)}}$$

$$e) \begin{array}{c} 15u^2 \\ 3u \cdot 5u \end{array} - 17uv \begin{array}{c} \underline{-42v^2} \\ \text{(-7v)(+6v)} \end{array} = \underline{\underline{(3u-7v)(5u+6v)}}$$

$$f) \begin{array}{c} 14x^2 \\ 2x \cdot 7x \end{array} - 41xy \begin{array}{c} \underline{+15y^2} \\ \text{(-3y)(-5y)} \\ \text{(-5y)(-3y)} \end{array} = \underline{\underline{(2x-5y)(7x-3y)}}$$

$$38. a) y^2 \begin{array}{c} \underline{-(c-3)} \\ \text{(-c)(+3)=-(-c-3)} \end{array} y \begin{array}{c} \underline{-3c} \\ \text{(+c)(-3)} \\ \text{(-c)(+3)} \end{array} = \underline{\underline{(y-c)(y+3)}}$$

$$b) z^2 + \begin{array}{c} \underline{(w-4)} \\ \text{(+w)(-4)} \end{array} z \begin{array}{c} \underline{-4w} \\ \text{(+w)(-4)} \\ \text{(-w)(+4)} \end{array} = \underline{\underline{(z+w)(z-4)}}$$

$$c) z^2 \begin{array}{c} \underline{-(5+p)} \\ \text{(-p)(-5)=-(-5+p)} \end{array} z \begin{array}{c} \underline{+5p} \\ \text{(-p)(-5)} \\ \text{(+p)(+5)} \end{array} = \underline{\underline{(z-p)(z-5)}}$$

$$d) u^2 \begin{array}{c} \underline{-(2t-5)} \\ \text{(-2t)(+5)=-(-2t-5)} \end{array} u \begin{array}{c} \underline{-10t} \\ \text{(+2t)(-5)} \\ \text{(-2t)(+5)} \end{array} = \underline{\underline{(u-2t)(u+5)}}$$

$$39. a) \Delta^2 + (a+b)\Delta + a \cdot b = (\Delta+a)(\Delta+b)$$

$$\begin{array}{c} (r-t)^2 \\ \Delta^2 \end{array} + \begin{array}{c} \underline{+8(r-t)} \\ \text{(a+b)\Delta} \end{array} + \begin{array}{c} \underline{+15} \\ ab \end{array} = [(r-t)+3][(r-t)+5] = \underline{\underline{(r-t+3)(r-t+5)}}$$

$$b) (-1)[(p+q)^2 - (p+q) - 12] = (-1)[(p+q)-4][(p+q)+3] =$$

$$\underline{\underline{(-1)(p+q-4)(p+q+3)}} = \underline{\underline{(4-p-q)(p+q+3)}}$$

$$c) (-1) \left[\begin{array}{cc} (d-e)^2 & -4 \\ \text{(-11)(+7)} & \end{array} \begin{array}{cc} (d-e) & -77 \\ \text{(+7)(-11)} & \end{array} \right] = (-1)[(d-e)+7][(d-e)-11] =$$

$$\underline{\underline{(d-e+7)(-d+e+11)}}$$

$$d) (m-n)^2 + 2(m-n) - 143 = \frac{(+13)(-11)}{(m-n)+(-11)} = \underline{\underline{(m-n+13)(m-n-11)}}$$

$$e) \left[\underbrace{3(m+n)}_{(3)(m+n)(3)(m+n)} \right]^2 - 12(m+n) \frac{-5}{\begin{matrix} (+5)(-1) \\ (-1)(+5) \\ (-5)(+1) \\ (+1)(-5) \end{matrix}} = [3(m+n) - 5][3(m+n) + 1] =$$

$$\underline{\underline{(3m + 3n - 5)(3m + 3n + 1)}}$$

oder

$$\left[\underbrace{3(m+n)}_{\Delta^2} \right]^2 - 4 \underbrace{[3(m+n)]}_{-4\Delta} \frac{-5}{(-5)(+1)} = [3(m+n) - 5][3(m+n) + 1] =$$

$$\underline{\underline{(3m + 3n - 5)(3m + 3n + 1)}}$$

$$f) (-1) \left[\frac{16(f+g)^2 - 24(f+g) - 7}{\begin{matrix} (+4)(+4) \\ (+7)(-1) \\ (-1)(+7) \\ (-7)(+1) \\ (+1)(-7) \end{matrix}} \right] = (-1)[4(f+g) - 7][4(f+g) + 1] =$$

$$\underline{\underline{(-1)(4f + 4g - 7)(4f + 4g + 1) = (7 - 4f - 4g)(4f + 4g + 1)}}$$

oder

$$(-1) \left\{ \underbrace{[4(f+g)]^2}_{\Delta^2} - 6 \cdot \underbrace{[4(f+g)]}_{-6\Delta} \frac{-7}{(-7)(+1)} \right\} = \underline{\underline{(-1)(4f + 4g - 7)(4f + 4g + 1) = (7 - 4f - 4g)(4f + 4g + 1)}}$$