5 Rationale Zahlen

5.21 Übungen Frommenwiler

72. b)

$$v-(v+z)\frac{\frac{v+z}{z-v}+\frac{4vz}{(v-z)(v+z)}}{\frac{v-z}{z}} = v-\frac{\frac{(-1)(v+z)(v+z)}{(-1)(z-v)}+\frac{\cancel{(v+z)}4vz}{(v-z)\cancel{(v+z)}}}{\frac{v-z}{z}}=$$

$$v-\frac{\frac{(-1)(v+z)^2+4vz}{(v-z)}}{\frac{v-z}{z}} = v-\frac{(-1)(v^2+2vz+z^2)+4vz}{v-z}\cdot\frac{z}{v-z}=$$

$$v-\frac{(-1)(v^2+2vz+z^2)+4vz}{(v-z)^2}\cdot z = v-\frac{-v^2-2vz-z^2+4vz}{(v-z)^2}\cdot z=$$

$$v-\frac{-v^2+2vz-z^2}{(v-z)^2}\cdot z = v-\frac{-\cancel{(v^2-2vz+z^2)}}{\cancel{(v-z)^2}}\cdot z = v-(-z) = \underline{\underline{v+z}}$$

73. a)

$$\frac{4a+3}{a+1}:\frac{2a+1-\frac{1}{4(2a+1)}}{\frac{(2a+1)^2+a}{2a+1}} = \frac{4a+3}{a+1}:\left[\frac{\overbrace{4(2a+1)^2}^{\text{quadratisch}}-1}{4\cancel{(2a+1)}}\cdot\frac{\cancel{2a+1}}{(2a+1)^2+a}\right]=$$

$$\frac{4a+3}{a+1}:\frac{\overbrace{[2(2a+1)-1][2(2a+1)+1]}^{\text{Binom}}}{4[(2a+1)^2+a]} = \frac{\cancel{4a+3}}{a+1}\cdot\frac{4[(2a+1)^2+a]}{(4a+1)\cancel{(4a+3)}}=$$

$$\frac{4(4a^2+4a+1+a)}{(a+1)(4a+1)}=\frac{4(4a^2+5a+1)}{(a+1)(4a+1)}=\frac{4\cancel{(4a+1)}\cancel{(a+1)}}{\cancel{(a+1)}\cancel{(4a+1)}}=\underline{\underline{4}}$$

b)

$$\frac{a(a-b)+b(a-b)-a^2}{\cancel{a}(a-b)}\cdot\frac{\cancel{2}\cancel{a}(b-a)}{\cancel{2}b^2(b-c)^2}=$$

$$\frac{(a^2-ab+ab-b^2-a^2)(b-a)}{(a-b)b^2(b-c)^2}=\frac{\cancel{-b^2}\cancel{(b-a)}}{\cancel{(-b^2)}\cancel{(-a+b)}(b-c)^2}=\underline{\underline{\frac{1}{(b-c)^2}}}$$

c)

$$\frac{(x-2)^3+x^3-8}{x-2}=\frac{(x-2)^3+\overbrace{x^3-2^3}^{(x-2)(x^2+2x+4)}}{x-2}=$$

$$\frac{\cancel{(x-2)}\left[(x-2)^2+(x^2+2x+4)\right]}{\cancel{x-2}}=$$

$$(x^2-4x+4)+(x^2+2x+4)=2x^2-2x+8=\underline{\underline{2(x^2-x+4)}}$$

oder (über Polynomdivision)

$$\frac{(x-2)^3+x^3-8}{x-2}=\frac{(x-2)^3}{x-2}+\frac{x^3-8}{x-2}=(x-2)^2+x^2+2x+4=$$

$$x^2-4x+4+x^2+2x+4=2x^2-2x+8=\underline{\underline{2(x^2-x+4)}}$$

Polynomdivision von: $\frac{x^3-8}{x-2}$

$$(x^3-8):(x-2)=\underline{x^2+2x+4}$$

$$\begin{array}{l} \underline{\overset{-}{+}x^3\overset{+}{-}2x^2} \\ \quad 2x^2-8 \\ \quad \underline{\overset{-}{+}2x^2\overset{+}{-}4x} \\ \qquad 4x-8 \\ \qquad \underline{\overset{-}{+}4x\overset{+}{-}8} \\ \qquad\quad 0 \end{array}$$

d)

$$\frac{\frac{1-7a}{(a-3)(a-1)}-\frac{a(a-1)}{(a-3)(a-1)}}{\frac{6a-(1-a^2)}{(1-a^2)6a}}=\frac{(1-7a)-a(a-1)}{(a-3)(a-1)}\cdot\frac{(1-a^2)6a}{6a-1+a^2}=$$

$$\frac{1-7a-a^2+a}{(a-3)(-1)\cancel{(-a+1)}}\cdot\frac{\cancel{(1-a)}(1+a)6a}{6a-1+a^2}=\frac{\overbrace{-a^2+1-6a}^{-(6a-1+a^2)}}{(a-3)(-1)}\cdot\frac{(1+a)6a}{6a-1+a^2}=$$

$$=\frac{-(6a-1+a^2)}{(a-3)(-1)}\cdot\frac{(1+a)(6a)}{6a-1+a^2}=\underline{\underline{\frac{6a(a+1)}{a-3}}}$$

74. d)

$$\frac{\left(\frac{b-a-a}{b-a}\right)^2}{\frac{\frac{1}{b-a}}{a}-\left(\frac{\frac{1}{a-b}}{a}\right)^2}=\frac{\left(\frac{b-2a}{b-a}\right)^2}{\frac{a}{b-a}\cdot\frac{-1}{-1}-\left(\frac{a}{a-b}\right)^2}=\frac{\left(\frac{b-2a}{b-a}\right)^2}{\frac{-a}{a-b}-\left(\frac{a}{a-b}\right)^2}=\frac{\left[\frac{b-2a}{-(a-b)}\right]^2}{\frac{-a}{a-b}-\left(\frac{a}{a-b}\right)^2}=$$

$$\frac{\frac{(b-2a)^2}{(a-b)^2}}{\frac{-a(a-b)-a^2}{(a-b)^2}}=\frac{(b-2a)^2}{(a-b)^2}\cdot\frac{(a-b)^2}{-a^2+ab-a^2}=\frac{(b-2a)^2}{-2a^2+ab}=\frac{(b-2a)^2}{a(b-2a)}=\underline{\underline{\frac{b-2a}{a}}}$$

75. d) $$\frac{\left(\frac{q^2-p^2}{pq}\right)^2(pq)^2}{(p+q)(q-p)}=\frac{\frac{[(q+p)(q-p)]^2}{(pq)^2}\cdot(pq)^2}{(p+q)(q-p)}=\frac{(q+p)^2(q-p)^2}{(p+q)(q-p)}=\underline{\underline{q^2-p^2}}$$

76. c) $$\left[p^2\frac{\overbrace{(1+p)^2-(1-p)^2}^{\text{Binom}}}{(1-p)(1+p)}\right]:\left[\frac{1+p-1}{1+p}\cdot\frac{\overbrace{1+p-(1-p)}^{1+p-1+p}}{1-p}\right]=$$

$$\frac{p^2\,\overbrace{(1+p+1-p)}^{2}\overbrace{(1+p-1+p)}^{2p}}{(1-p)(1+p)}\cdot\frac{(1+p)(1-p)}{p\cdot 2p}=\frac{2\cdot 2p}{2}=\underline{\underline{2p}}$$

d) $$\frac{(a-b)^2\left[\overbrace{(a+b)^2+2(a+b)(a-b)+(a-b)^2}^{\text{Binom}}\right]}{a\left(\underbrace{a^2-2ab+b^2}_{\text{Binom}}\right)\left[\underbrace{(a+b)^2-(a-b)^2}_{\text{Binom}}\right]}=$$

$$\frac{(a-b)^2[(a+b)+(a-b)]^2}{a(a-b)^2(a+b+a-b)(a+b-a+b)}=\frac{(a-b)^2[2a]^2}{a(a-b)^2(2a)(2b)}=\frac{2a}{2ab}=\underline{\underline{\frac{1}{b}}}$$

77. b)

$$\frac{1}{b(abc+a+c)}-\frac{\frac{ab+1}{b}}{\frac{abc+a+c}{bc+1}}=\frac{1}{b(abc+a+c)}-\frac{ab+1}{b}\cdot\frac{bc+1}{abc+a+c}=$$

$$\frac{1-(ab+1)(bc+1)}{b(abc+a+c)}=\frac{1-(ab^2c+ab+bc+1)}{b(abc+a+c)}=$$

$$\frac{\cancel{1}-ab^2c-ab-bc\cancel{-1}}{b(abc+a+c)}=\frac{-b\cancel{(abc+a+c)}}{b\cancel{(abc+a+c)}}=\underline{\underline{-1}}$$

78. b)

$$\frac{1}{\frac{1+2z+z^2}{1+2z}}-\frac{\frac{(z+1.5)(1-z)+(z+0.5)(1+z)}{(1+z)(1-z)}}{\frac{2}{z}-\frac{1}{\frac{3z^3-z-z^2(3z-1)}{3z-1}}}=$$

$$\frac{1+2z}{(z+1)^2}-\frac{\frac{z\cancel{-z^2}+1.5-1.5z+z\cancel{+z^2}+0.5+0.5z}{(1+z)(1-z)}}{\frac{2}{z}-\frac{3z-1}{\cancel{3z^3}-z\cancel{-3z^3}+z^2}}=$$

$$\frac{1+2z}{(z+1)^2}-\frac{\frac{z+2}{(1+z)(1-z)}}{\frac{2}{z}-\frac{3z-1}{z(z-1)}}=\frac{1+2z}{(z+1)^2}-\frac{\frac{z+2}{(1+z)(1-z)}}{\frac{2(z-1)-3z+1}{z(z-1)}}=$$

$$\frac{1+2z}{(z+1)^2}-\frac{z+2}{(1+z)\cancel{(1-z)}}\cdot\frac{-z\cancel{(1-z)}}{\underbrace{2z-2-3z+1}_{-z-1}}=\frac{1+2z}{(z+1)^2}-\frac{z+2}{(1+z)}\cdot\frac{-z}{-(1+z)}=$$

$$\frac{1+2z}{(z+1)^2}-\frac{z+2}{(1+z)}\cdot\frac{z}{(1+z)}=\frac{1+2z-z(z+2)}{(z+1)^2}=$$

$$\frac{1+2z-z^2-2z}{(z+1)^2}=\frac{1-z^2}{(z+1)^2}=\frac{(1-z)\cancel{(1+z)}}{(z+1)^{\cancel{2}}}=\underline{\underline{\frac{1-z}{z+1}}}$$