

5 Rationale Zahlen

5.9 Übungen Frommenwiler

$$53. \quad a) \quad \frac{u \cdot 2}{(u+v) \cdot 2} - \frac{u-v}{2(u+v)} = \frac{2u-u+v}{2(u+v)} = \frac{\cancel{u+v}}{2(\cancel{u+v})} = \frac{1}{2}$$

$$b) \quad \frac{(m-n)(m-n)}{(m+n)(m-n)} - \frac{(m+n)(m+n)}{(m-n)(m+n)} =$$

$$\frac{m^2 - 2mn + n^2 - (m^2 + 2mn + n^2)}{(m-n)(m+n)} =$$

$$\frac{m^2 - 2mn + n^2 - m^2 - 2mn - n^2}{(m-n)(m+n)} = \frac{-4mn}{m^2 - n^2} = \frac{4mn}{n^2 - m^2}$$

oder

$$\frac{\overbrace{(m-n)^2 - (m+n)^2}^{3. \text{ Binom}}}{(m+n)(m-n)} = \frac{(m-n+m+n)(m-n-m-n)}{(m+n)(m-n)} = \frac{2m \cdot (-2n)}{m^2 - n^2} = \frac{4mn}{n^2 - m^2}$$

$$d) \quad \frac{m}{m-n} - \frac{n(-1)}{(n-m)(-1)} - \frac{2mn}{(m-n)(m+n)} =$$

$$\frac{m}{m-n} - \frac{-n}{m-n} - \frac{2mn}{(m-n)(m+n)} =$$

$$\frac{m(m+n) + n(m+n) - 2mn}{(m-n)(m+n)} = \frac{m^2 + mn + mn + n^2 - 2mn}{(m-n)(m+n)} = \frac{m^2 + n^2}{m^2 - n^2}$$

e) konsequent mit Binomen gerechnet:

$$\frac{(3x-2)^2}{(2-3x)(2+3x)} + \frac{\overbrace{\cancel{3} \cdot (9x^2 - 4)}^{3. \text{ Binom}}}{\underbrace{\cancel{3} \cdot (9x^2 - 12x + 4)}_{2. \text{ Binom}}} =$$

$$\frac{(-1)(3x-2)^2}{(-2+3x)(2+3x)} + \frac{\cancel{(3x-2)} \cdot (3x+2)}{(3x-2)^2} \cdot \frac{\overbrace{3x+2}^{\text{erweitert}}}{3x+2} = \quad 3. \text{ Binom im Zähler (als Summe)}$$

$$\frac{(3x+2)^2 - (3x-2)^2}{(3x-2)(3x+2)} =$$

$$\frac{(3x+2+3x-2)(3x+2-3x+2)}{(3x-2)(3x+2)} = \frac{6x \cdot 4}{(3x-2)(3x+2)} = \frac{24x}{9x^2 - 4}$$

3. Binom im Zähler wird nicht erkannt:

$$\frac{(3x-2)^2}{(2-3x)(2+3x)} + \frac{\cancel{3}(9x^2-4)}{\cancel{3}(9x^2-12x+4)} =$$

$$\frac{(-1)(3x-2)^2}{(-2+3x)(2+3x)} + \frac{\cancel{(3x-2)}(3x+2)}{(3x-2)^2} \cdot \frac{\overset{\text{erweitert}}{3x+2}}{3x+2} =$$

$$\frac{(-1)(3x-2)^2 + (3x+2)^2}{(2+3x)(3x-2)} = \quad \text{3. Binom wird nicht erkannt}$$

$$\frac{(-1)(9x^2-12x+4) + 9x^2 + 12x + 4}{(3x+2)(3x-2)} = \frac{-9x^2 + 12x - 4 + 9x^2 + 12x + 4}{(3x+2)(3x-2)} = \frac{24x}{\underline{\underline{9x^2-4}}}$$

54. a) $\frac{m+13}{(m+3)(m-2)} - \frac{m+1}{(m-2)(m-1)} =$

$$\frac{(m+13)(m-1)}{(m+3)(m-2)(m-1)} - \frac{(m+3)(m+1)}{(m+3)(m-2)(m-1)} =$$

$$\frac{m^2 - m + 13m - 13 - (m^2 + m + 3m + 3)}{(m+3)(m-2)(m-1)} =$$

$$\frac{m^2 - m + 13m - 13 - m^2 - m - 3m - 3}{(m+3)(m-2)(m-1)} =$$

$$\frac{8m - 16}{(m+3)(m-2)(m-1)} = \frac{8\cancel{(m-2)}}{(m+3)\cancel{(m-2)}(m-1)} = \frac{8}{\underline{\underline{(m+3)(m-1)}}$$

b) $\frac{x+5}{(x-3)(x+1)} + \frac{9x-7}{(x-3)(x+2)} - \frac{4x+3}{(x+2)(x+1)} =$

$$\frac{(x+5)(x+2)}{(x-3)(x+2)(x+1)} + \frac{(9x-7)(x+1)}{(x-3)(x+2)(x+1)} - \frac{(x-3)(4x+3)}{(x-3)(x+2)(x+1)} =$$

$$\frac{x^2 + 7x + 10 + 9x^2 + 2x - 7 - (4x^2 - 9x - 9)}{(x-3)(x+2)(x+1)} =$$

$$\frac{x^2 + 7x + 10 + 9x^2 + 2x - 7 - 4x^2 + 9x + 9}{(x-3)(x+2)(x+1)} =$$

$$\frac{6x^2 + 18x + 12}{(x-3)(x+2)(x+1)} = \frac{6(x^2 + 3x + 2)}{(x-3)(x+2)(x+1)} = \frac{6\cancel{(x+1)}\cancel{(x+2)}}{(x-3)\cancel{(x+2)}\cancel{(x+1)}} = \frac{6}{\underline{\underline{x-3}}}$$

$$\begin{aligned}
 \text{d) } & \frac{n+1}{2(n-1)} + \frac{\cancel{n}(2n-3)}{\cancel{n}(n+1)} + \frac{n^2+3}{-2(n^2-1)} = \\
 & \frac{n+1}{2(n-1)} + \frac{2n-3}{n+1} + \frac{(n^2+3)(-1)}{-2(n-1)(n+1)(-1)} = \\
 & \frac{(n+1)(n+1)}{2(n-1)(n+1)} + \frac{2(2n-3)(n-1)}{2(n+1)(n-1)} + \frac{(n^2+3)(-1)}{2(n+1)(n-1)} = \\
 & \frac{n^2+2n+1+2(2n^2-5n+3)+(-n^2-3)}{2(n+1)(n-1)} = \\
 & \frac{n^2+2n+1+4n^2-10n+6-n^2-3}{2(n+1)(n-1)} = \\
 & \frac{4n^2-8n+4}{2(n+1)(n-1)} = \frac{\cancel{2}(n^2-2n+1)}{\cancel{2}(n+1)(n-1)} = \frac{2(n-1)^{\cancel{2}}}{(n+1)\cancel{(n-1)}} = \underline{\underline{\frac{2(n-1)}{n+1}}}
 \end{aligned}$$