9 Gleichungen

9.17 Übungen Frommenwiler (noch korrigieren)

324. a) $\frac{45}{2y-9}-2=-\frac{27}{9-2y}\cdot\frac{-1}{-1}$

$2y-9\neq 0 \rightarrow 2y\neq 9 \rightarrow y\neq\frac{9}{2} \rightarrow D=\mathbb{R}\setminus\left\{\frac{9}{2}\right\}$

$\frac{45}{2y-9}-2=\frac{27}{2y-9} \quad \left|-\frac{27}{2y-9}+2\right.$

$\frac{45}{2y-9}-\frac{27}{2y-9}=2$

$\frac{45-27}{2y-9}=2 \quad \left|\cdot(2y-9)\right.$

$18=2\cdot(2y-9)$

$18=4y-18 \quad \left|+18\right.$

$36=4y \quad \left|:4\right.$

$y=\underline{9\in D}$

Kontrolle: $\underbrace{\frac{45}{2\cdot 9-9}}_{3}-2=\underbrace{-\frac{27}{9-2\cdot 9}}_{3}$ (w)

somit: $L=\underline{\underline{\{9\}}}$

b) $\frac{6x+5}{2}+\frac{4x-13}{11x+1}=3x$

$11x+1\neq 0 \quad\rightarrow\quad 11x\neq -1 \quad\rightarrow\quad x\neq -\frac{1}{11} \quad\rightarrow\quad D=\mathbf{R}\setminus\left\{-\frac{1}{11}\right\}$

$\frac{6x+5}{2}\cdot\frac{11x+1}{11x+1}+\frac{4x-13}{11x+1}\cdot\frac{2}{2}=3x\cdot\frac{11x+1}{11x+1}\cdot\frac{2}{2}$ | ·HN

$66x^2+6x+55x+5+8x-26=66x^2+6x$ | $-66x^2$

$69x-21=6x$ | $-6x+21$

$63x=21$ | $:63$

$x=\frac{21}{63}=\underline{\frac{1}{3}\in D}$

Kontrolle: $\underbrace{\frac{6\cdot\frac{1}{3}+5}{2}+\frac{4\cdot\frac{1}{3}-13}{11\cdot\frac{1}{3}+1}}_{1}=3\cdot\frac{1}{3}\,(w)$ $_1$

somit: $\underline{\underline{L=\left\{\frac{1}{3}\right\}}}$

c) $\frac{4z+6}{2z-10}-\frac{6z-43}{5-z}=124$

$5-z\neq 0 \quad\rightarrow\quad z\neq 5 \quad\rightarrow\quad D=\mathbf{R}\setminus\{5\}$

$\frac{2(2z+3)}{-2(5-z)}-\frac{6z-43}{5-z}=124$ | kürzen

$\frac{-(2z+3)-6z+43}{5-z}=124$ | $\cdot(5-z)$

$-2z-3-6z+43=124(5-z)=620-124z$ | zusammenfassen

$-8z+40=620-124z$ | $+124z-40$

$116z=580$ | $:116$

$z=\frac{580}{116}=\underline{5\notin D}$

somit: $\underline{\underline{L=\{\ \}}}$

e) $-\dfrac{5}{\dfrac{x-5}{7}} = -3 + \dfrac{\dfrac{7x}{2}}{\dfrac{5}{2} - \dfrac{x}{2}}$

$x-5 \neq 0 \quad \wedge \quad 5-x \neq 0 \quad \rightarrow \quad D = \mathbb{R} \setminus \{5\}$

$-\dfrac{5}{\dfrac{x-5}{7}} \cdot \dfrac{7}{7} = -3 + \dfrac{\dfrac{7x}{2}}{\dfrac{5}{2} - \dfrac{x}{2}} \cdot \dfrac{2}{2}$ | erweitern

$-\dfrac{35}{x-5} = -3 + \dfrac{7x}{5-x}$ | Minus in Nenner

$\dfrac{35}{5-x} = -3 + \dfrac{7x}{5-x}$ | $\cdot(5-x)$

$35 = -3(5-x) + 7x$

$35 = -15 + 3x + 7x$

$50 = 10x$

$x = \underline{5 \notin D}$

somit: $L = \underline{\underline{\{\ \}}}$

326. a) $\frac{1}{x+1}+\frac{x}{x+2}+\underbrace{\frac{\cancel{x+3}}{(x+3)^{\cancel{2}}}}_{\text{weil } x=-3 \text{ keine gültige Lösung ist!}}=1$ $\quad|\cdot HN$

$D=\mathbf{R}\setminus\{-1;-2;-3\}$

$(x+2)(x+3)+x(x+1)(x+3)+(x+1)(x+2)=(x+1)(x+2)(x+3)$

$x^2+5x+6+x\left(x^2+4x+3\right)+x^2+3x+2=(x+1)\left(x^2+5x+6\right)$

$x^2+5x+6+x^3+4x^2+3x+x^2+3x+2=x^3+5x^2+6x+x^2+5x+6$ $\quad|$ordnen

$x^3+6x^2+11x+8=x^3+6x^2+11x+6$ $\quad|-x^3-6x^2-11x$

$8=6$ (f)

$L=\{\ \}$

b) $\frac{1-4t}{(t-2)(t-1)}-\frac{3(t-1)}{(t-6)(t-2)}+\frac{7\cancel{t}}{\cancel{t}(t-6)}=0$ $\quad|\cdot HN$

$D=\mathbf{R}\setminus\{0;1;2;6\}$

$(1-4t)(t-6)-3(t-1)^2+7(t-1)(t-2)=0$ $\quad|$ausmult.

$\left(25t-6-4t^2\right)-3\left(t^2-2t+1\right)+7\left(t^2-3t+2\right)=0$

$25t-6-4t^2-3t^2+6t-3+7t^2-21t+14=0$

$10t+5=0$ $\quad|:5$

$2t+1=0$ $\quad|-1$

$2t=-1$ $\quad|:2$

$t=-\frac{1}{2}\in D$

Kontrolle: $\underbrace{\underbrace{\frac{1-4\left(-\frac{1}{2}\right)}{\left(-\frac{1}{2}\right)^2-3\left(-\frac{1}{2}\right)+2}}_{\frac{4}{5}}-\underbrace{\frac{3\left(-\frac{1}{2}-1\right)}{\left(-\frac{1}{2}\right)^2-8\left(-\frac{1}{2}\right)+12}}_{-\frac{18}{65}}+\underbrace{\frac{7\left(-\frac{1}{2}\right)}{\left(-\frac{1}{2}\right)^2-6\left(-\frac{1}{2}\right)}}_{-\frac{14}{13}}}_{\text{mit Taschenrechner: 0}}=0\ (w)$

somit: $L=\left\{-\frac{1}{2}\right\}$

. c) $\frac{3x+4}{x-5}-\frac{x-9}{x-7}=\frac{2x^2-13x+27}{(x-5)(x-7)}$ $\mid\cdot HN$

$D=\mathbf{R}\setminus\{5;7\}$

$(3x+4)(x-7)-(x-9)(x-5)=2x^2-13x+27$ $\mid$ausmult.

$3x^2-17x-28-\left(x^2-14x+45\right)=2x^2-13x+27$

$3x^2-17x-28-x^2+14x-45=2x^2-13x+27$

$2x^2-3x-73=2x^2-13x+27$ $\mid -2x^2+13x+73$

$10x=100$

$x=\underline{10}\in D$

Kontrolle: $\underbrace{\frac{3\cdot 10+4}{10-5}-\frac{10-9}{10-7}}_{\frac{97}{15}}=\underbrace{\frac{2\cdot 10^2-13\cdot 10+27}{10^2-12\cdot 10+35}}_{\frac{97}{15}}(w)$

somit: $L=\underline{\underline{\{10\}}}$

d) $\frac{3}{z+3}-\frac{3}{z+8}=\frac{5}{z-8}-\frac{5}{z-5}$ |gleichn.

$D=\mathbf{R}\setminus\{-8;-3;5;8\}$

$\frac{3(z+8)-3(z+3)}{(z+3)(z+8)}=\frac{5(z-5)-5(z-8)}{(z-8)(z-5)}$ |ausmult.

$\frac{3z+24-3z-9}{(z+3)(z+8)}=\frac{5z-25-5z+40}{(z-8)(z-5)}$ |zusammenf.

$\frac{15}{(z+3)(z+8)}=\frac{15}{(z-8)(z-5)}$ $|N_1=N_2$

$(z+3)(z+8)=(z-8)(z-5)$

$z^2+11z+24=z^2-13z+40$ $|-z^2+13z-24$

$24z=16$ $|:24$

$z=\frac{16}{24}=\underline{\frac{2}{3}}\in D$

Kontrolle: $\underbrace{\frac{3}{\frac{2}{3}+3}-\frac{3}{\frac{2}{3}+8}}_{\frac{135}{286}}=\underbrace{\frac{5}{\frac{2}{3}-8}-\frac{5}{\frac{2}{3}-5}}_{\frac{135}{286}}$ (w)

somit: $\underline{\underline{L=\left\{\frac{2}{3}\right\}}}$

oder

$\frac{3}{z+3}-\frac{3}{z+8}=\frac{5}{z-8}-\frac{5}{z-5}$ |auskl.|gleichn.

$D=\mathbf{R}\setminus\{-8;-3;5;8\}$

$\frac{3(z+8-z-3)}{(z+3)(z+8)}=\frac{5(z-5-z+8)}{(z-8)(z-5)}$ |zusammenf.

$\frac{3\cdot 5}{(z+3)(z+8)}=\frac{5\cdot 3}{(z-8)(z-5)}$ $|N_1=N_2$

$(z+3)(z+8)=(z-8)(z-5)$

Fortsetzung siehe oben!

e) $\dfrac{3}{1+\dfrac{1}{p}}\cdot\dfrac{p}{p}-\dfrac{1+\dfrac{1}{p}}{1-\dfrac{1}{p}}\cdot\dfrac{p}{p}=2$ | Brüche weg

$D=\mathbf{R}\setminus\{-1;0;1\}$

$\dfrac{3p}{p+1}-\dfrac{p+1}{p-1}=2$ | ·HN

$3p(p-1)-(p+1)(p+1)=2(p+1)(p-1)$ | ausmulti.

$3p^2-3p-(p^2+2p+1)=2(p^2-1)$

$3p^2-3p-p^2-2p-1=2p^2-2$ | zusammenf.

$2p^2-5p-1=2p^2-2$ | $-2p^2+1$

$-5p=-1$ | $:(-5)$

$p=\dfrac{-1}{-5}=\underline{\dfrac{1}{5}}\in D$

Kontrolle: $\underbrace{\dfrac{3}{1+5}-\dfrac{1+5}{1-5}}_{2}=2\ (w)$

somit: $L=\underline{\underline{\left\{\dfrac{1}{5}\right\}}}$

f) $\dfrac{\frac{2}{0.9}-\frac{2}{y}}{\frac{1}{0.9}+\frac{1}{y}}=\frac{2}{3}$

$y\neq \underline{0} \quad \wedge \quad \frac{10}{9}+\frac{1}{y}\neq 0 \rightarrow \frac{1}{y}\neq -\frac{10}{9} \rightarrow y\neq -\underline{\frac{9}{10}} \quad \rightarrow \quad D=\mathbb{R}\setminus\left\{-\frac{9}{10};0\right\}$

$\dfrac{\frac{2\cdot 10}{9}-\frac{2}{y}}{\frac{1\cdot 10}{9}+\frac{1}{y}}\cdot\frac{9y}{9y}=\frac{2}{3}$ | Brüche weg

$\frac{20y-18}{10y+9}=\frac{2}{3}$

$3(20y-18)=2(10y+9)$ | ausmulti.

$60y-54=20y+18$ | $-20y+54$

$40y=72$ | $:40$

$y=\frac{72}{40}=\underline{\frac{9}{5}}\in D$

Kontrolle: $\underbrace{\dfrac{\frac{2}{0.9}-\frac{2\cdot 5}{9}}{\frac{1}{0.9}+\frac{5}{9}}}_{\frac{2}{3}}=\frac{2}{3}\,(w)$

somit: $L=\underline{\underline{\left\{\frac{9}{5}\right\}}}$