

9 Gleichungen

9.21 Übungen Frommenwiler (noch korrigieren)

$$328. \text{ b) } \left(r - \frac{s}{1+x}\right)(p+q) = \left(r + \frac{s}{1+x}\right)(p-q)$$

$$D = \mathbb{R} \setminus \{-1\}$$

$$\left(r - \frac{s}{1+x}\right)(p+q) = \left(r + \frac{s}{1+x}\right)(p-q) \quad | \cdot (1+x)$$

$$[r(1+x) - s](p+q) = [r(1+x) + s](p-q) \quad | \text{ausmulti.}$$

$$r(1+x)(p+q) - s(p+q) = r(1+x)(p-q) + s(p-q) \quad | \text{TU}$$

$$r(1+x)(p+q) - r(1+x)(p-q) = s(p-q) + s(p+q)$$

$$r(1+x)[p+q-p+q] = s(p-q+p+q)$$

$$r(1+x)(2q) = 2ps \quad | : (2qr)$$

$$1+x = \frac{2ps}{2qr} = \frac{ps}{qr} \quad | -1$$

$$x = \frac{ps}{qr} - 1 \in D \quad \underline{q \neq 0} \quad \wedge \quad \underline{r \neq 0}$$

$$L = \left\{ x \in \mathbb{R} \mid x = \frac{ps}{qr} - 1 \right\}$$

$$\text{d) } \frac{a - \frac{1}{x}}{a + \frac{1}{x}} - \frac{x - \frac{1}{a}}{x + \frac{1}{a}} = 0$$

$$\underline{x \neq 0} \quad \wedge \quad a + \frac{1}{x} \neq 0 \rightarrow \frac{1}{x} \neq -a \rightarrow \underline{x \neq -\frac{1}{a}} \quad \wedge \quad x + \frac{1}{a} \neq 0 \rightarrow \underline{x \neq -\frac{1}{a}}$$

$$D = \mathbb{R} \setminus \left\{ -\frac{1}{a}; 0 \right\}$$

$$\frac{a - \frac{1}{x}}{a + \frac{1}{x}} \cdot \frac{x}{x} - \frac{x - \frac{1}{a}}{x + \frac{1}{a}} \cdot \frac{a}{a} = 0$$

|Brüche weg

$$\frac{ax - 1}{ax + 1} - \frac{ax - 1}{ax + 1} = 0$$

|·(ax + 1)

$$ax - 1 - ax + 1 = 0$$

$$0 = 0 \text{ (w)}$$

$$L = D = \underline{\underline{\mathbf{R} \setminus \left\{ -\frac{1}{a}; 0 \right\}}} \quad \underline{a \neq 0}$$

$$e) \quad \frac{bc}{x(a+b)} - \frac{a^2 + b^2}{2b(a+b)} = \frac{c(a-b)}{2bx} - \frac{a^2x - b^2c}{bx(a+b)}$$

$$\underline{x \neq 0} \quad \wedge \quad \underline{a \neq -b} \quad \wedge \quad \underline{b \neq 0}$$

$$D = \mathbf{R} \setminus \{0\}$$

$$\frac{bc}{x(a+b)} \cdot \frac{2b}{2b} - \frac{a^2 + b^2}{2b(a+b)} \cdot \frac{x}{x} = \frac{c(a-b)}{2bx} \cdot \frac{a+b}{a+b} - \frac{a^2x - b^2c}{bx(a+b)} \cdot \frac{2}{2}$$

|·HN

$$2b^2c - a^2x - b^2x = c(a^2 - b^2) - 2a^2x + 2b^2c$$

|+2a²x - 2b²c

$$a^2x - b^2x = a^2c - b^2c$$

|faktorisieren

$$x(a^2 - b^2) = c(a^2 - b^2)$$

|:(a² - b²)

$$x = \underline{c} \in D \quad \underline{a \neq b} \quad \wedge \quad \underline{a \neq -b}$$

$$L = \underline{\underline{\{x \in \mathbf{R} | x = c\}}}$$

339. a)

$$340. \text{ a) } 4 = 40 - 6\sqrt{y} \quad | -40$$

$$D = \{y \in \mathbf{R} \mid y \geq 0\}$$

$$4 - 40 = -6\sqrt{y} \quad | :(-6)$$

$$\frac{-36}{-6} = \sqrt{y} \quad | ()^2$$

$$y = \underline{\underline{36}} \in D$$

$$\text{Kontrolle: } 4 = \underbrace{40 - 6\sqrt{36}}_4 \quad (\text{w})$$

$$\text{somit: } L = \underline{\underline{\{36\}}}$$

$$\text{b) } 4 + \sqrt{100z} = 24 \quad | -4$$

$$D = \{z \in \mathbf{R} \mid z \geq 0\}$$

$$\sqrt{100z} = 20 \quad | ()^2$$

$$100z = 400 \quad | :100$$

$$z = \underline{\underline{4}} \in D$$

$$\text{Kontrolle: } 4 + \underbrace{\sqrt{100 \cdot 4}}_{10 \cdot 2} = 24 \quad (\text{w})$$

$$\text{somit: } L = \underline{\underline{\{4\}}}$$

$$\text{c) } 21 = 6 + 3\sqrt{-u} \quad | -6$$

$$D = \{u \in \mathbf{R} \mid u \leq 0\}$$

$$15 = 3\sqrt{-u} \quad | :3$$

$$5 = \sqrt{-u} \quad | ()^2$$

$$25 = -u$$

$$u = \underline{\underline{-25}} \in D$$

$$\text{Kontrolle: } 21 = 6 + 3\underbrace{\sqrt{-(-25)}}_{6+3\sqrt{25}} \quad (\text{w})$$

$$\text{somit: } L = \underline{\underline{\{-25\}}}$$

$$d) \quad 2 + 2\sqrt{\frac{25}{t}} = 12 \quad | -2$$

$$t \neq 0 \quad \wedge \quad t \geq 0 \quad \rightarrow \quad D = \{t \in \mathbf{R} \mid t > 0\}$$

Nenner $\neq 0$ Radikand ≥ 0

$$2\sqrt{\frac{25}{t}} = 10 \quad | :2$$

$$\sqrt{\frac{25}{t}} = 5 \quad | ()^2$$

$$\frac{25}{t} = 25 \quad | \cdot t$$

$$25 = 25t \quad | :25$$

$$t = \underline{1} \in D$$

$$\text{Kontrolle: } 2 + 2\sqrt{\frac{25}{1}} = 12 \quad (w)$$

$$\text{somit: } L = \underline{\underline{\{1\}}}$$

$$e) \quad 6.5 - \sqrt{1+4w} = \frac{3}{2} \quad \left| -\frac{3}{2} + \sqrt{1+4w} \right.$$

$$1+4w \geq 0 \quad \rightarrow \quad 4w \geq -1 \quad \rightarrow \quad w \geq -\frac{1}{4} \quad \rightarrow \quad D = \left\{ w \in \mathbf{R} \mid w \geq -\frac{1}{4} \right\}$$

$$6.5 - \frac{3}{2} = \sqrt{1+4w} \quad | ()^2$$

$$5^2 = 1+4w \quad | -1$$

$$24 = 4w \quad | :4$$

$$w = \underline{6} \in D$$

$$\text{Kontrolle: } \underbrace{6.5 - \sqrt{1+4 \cdot 6}}_{1.5} = \frac{3}{2} \quad (w)$$

$$\text{somit: } L = \underline{\underline{\{6\}}}$$

$$f) \sqrt{19 + 2\sqrt{\frac{4x+4}{x-4}}} = 5$$

Definitionsbereich festlegen:

$$\text{Radikand} \geq 0: \sqrt{19 + 2\sqrt{\frac{4x+4}{x-4}}} = 5$$

$$\text{Fallunterscheidung: } \frac{+}{+} \geq 0 \quad \text{oder} \quad \frac{-}{-} \geq 0$$

1. Fall: $\frac{+}{+} \geq 0$	2. Fall: $\frac{-}{-} \geq 0$
$\left. \begin{array}{l} 4x+4 \geq 0 \quad \rightarrow x \geq -1 \\ x-4 > 0 \quad \rightarrow x > 4 \\ \text{=0} \\ \text{ist verboten} \end{array} \right\} \underline{x > 4}$	$\left. \begin{array}{l} 4x+4 \leq 0 \quad \rightarrow x \leq -1 \\ x-4 < 0 \quad \rightarrow x < 4 \\ \text{=0} \\ \text{ist verboten} \end{array} \right\} \underline{x \leq -1}$

$$\text{somit: } D = \left\{ x \in \mathbf{R} \mid x \leq -1 \quad \vee \quad x > 4 \right\}$$

oder

Lösung der Wurzelgleichung:	Kontrolle der Lösung:
$\sqrt{19 + 2\sqrt{\frac{4x+4}{x-4}}} = 5 \quad ()^2$	$\sqrt{19 + 2\sqrt{\frac{4 \cdot 8 + 4}{8 - 4}}} = 5$
$19 + 2\sqrt{\frac{4x+4}{x-4}} = 25 \quad -19$	$\sqrt{19 + 2\sqrt{\frac{36}{4}}} = 5$
$2\sqrt{\frac{4x+4}{x-4}} = 6 \quad :2$	$\sqrt{19 + 2 \cdot \frac{6}{2}} = 5$
$\sqrt{\frac{4x+4}{x-4}} = 3 \quad ()^2$	$\sqrt{19 + 6} = 5$
$\frac{4x+4}{x-4} = 9 \quad \cdot (x-4)$	$\sqrt{25} = 5$
$4x+4 = 9x-36 \quad -4x \quad +36$	$5 = 5 \quad (\text{w})$
$40 = 5x$	$\text{somit: } L = \underline{\underline{\{8\}}}$
$x = \underline{\underline{8}} \in D$	

$$341. \text{ a) } 8 - 3\sqrt{2u-1} = 2$$

$$2u-1 \geq 0 \rightarrow u \geq \frac{1}{2} \rightarrow D = \left\{ u \in \mathbf{R} \mid u \geq \frac{1}{2} \right\}$$

$$6 = 3\sqrt{2u-1} \quad | :3$$

$$2 = \sqrt{2u-1} \quad | ()^2$$

$$4 = 2u-1 \quad | +1$$

$$5 = 2u \quad | :2$$

$$u = \frac{5}{2} \in D$$

$$\text{Kontrolle: } \underbrace{8 - 3\sqrt{2 \cdot \frac{5}{2} - 1}}_2 = 2 \quad (\text{w})$$

$$\text{somit: } L = \underline{\underline{\left\{ \frac{5}{2} \right\}}}$$

$$\text{b) } \sqrt{x+5} - 3\sqrt{4-x} = 0$$

$$| +3\sqrt{4-x}$$

$$x+5 \geq 0 \quad \wedge \quad 4-x \geq 0$$

$$x \geq -5 \quad \wedge \quad x \leq 4 \rightarrow D = \{ x \in \mathbf{R} \mid -5 \leq x \leq 4 \}$$

$$\sqrt{x+5} = 3\sqrt{4-x}$$

$$| ()^2$$

$$x+5 = 9(4-x) = 36-9x$$

$$| +9x - 5$$

$$10x = 31$$

$$| :10$$

$$x = \underline{3.1} \in D$$

$$\text{Kontrolle: } \underbrace{\sqrt{3.1+5} - 3\sqrt{4-3.1}}_0 = 0 \quad (\text{w})$$

$$\text{somit: } L = \underline{\underline{\{3.1\}}}$$

$$c) \sqrt{a^2 + 7a + 6} - 3 = a \quad | +3$$

$$a^2 + 7a + 6 \geq 0 \rightarrow (a+6)(a+1) \geq 0$$

Fallunterscheidung: $(+) \cdot (+) \geq 0$ oder $(-) \cdot (-) \geq 0$

$$1. \text{ Fall } (+) \cdot (+) \geq 0: a \geq -6 \wedge a \geq -1 \rightarrow \underline{a \geq -1}$$

$$2. \text{ Fall } (-) \cdot (-) \geq 0: a \leq -6 \wedge a \leq -1 \rightarrow \underline{a \leq -6}$$

$$\text{somit: } D = \{a \in \mathbb{R} \mid a \leq -6 \vee a \geq -1\}$$

$$\sqrt{a^2 + 7a + 6} = a + 3 \quad | ()^2$$

$$\cancel{a^2} + 7a + 6 = \cancel{a^2} + 6a + 9 \quad | -6a - 6$$

$$a = \underline{3} \in D$$

$$\text{Kontrolle: } \underbrace{\sqrt{3^2 + 7 \cdot 3 + 6}}_6 - 3 = 3 \quad (w)$$

$$\text{somit: } L = \underline{\underline{\{3\}}}$$

$$d) 3\sqrt{y+2} + 2\sqrt{2y+11} = 9\sqrt{y+2} \quad | -3\sqrt{y+2}$$

$$y \geq -2 \wedge y \geq -\frac{11}{2} \wedge y \geq -2 \rightarrow D = \{y \in \mathbb{R} \mid y \geq -2\}$$

$$2\sqrt{2y+11} = 6\sqrt{y+2} \quad | :2$$

$$\sqrt{2y+11} = 3\sqrt{y+2} \quad | ()^2$$

$$2y+11 = 9(y+2) = 9y+18 \quad | -2y - 18$$

$$-7 = 7y$$

$$y = \underline{-1} \in D$$

$$\text{Kontrolle: } \underbrace{3\sqrt{-1+2}}_3 + \underbrace{2\sqrt{2 \cdot (-1) + 11}}_6 = \underbrace{9\sqrt{-1+2}}_9 \quad (w)$$

$$\text{somit: } L = \underline{\underline{\{-1\}}}$$

$$e) \quad 3\sqrt{3p-5} - 2 = \sqrt{12p-20} + 2$$

|faktorisieren

$$3p-5 \geq 0 \rightarrow 3p \geq 5 \rightarrow p \geq \frac{5}{3} \rightarrow D = \left\{ p \in \mathbf{R} \mid p \geq \frac{5}{3} \right\}$$

$$3\sqrt{3p-5} - 2 = \sqrt{4(3p-5)} + 2$$

|teilweise radizieren

$$3\sqrt{3p-5} - 2 = 2\sqrt{3p-5} + 2$$

$$|-2\sqrt{3p-5} + 2$$

$$\sqrt{3p-5} = 4$$

$$|(\)^2$$

$$3p-5 = 16$$

$$|+5$$

$$3p = 21$$

$$|:3$$

$$p = \underline{7} \in D$$

$$\text{Kontrolle: } \underbrace{3\sqrt{3 \cdot 7 - 5} - 2}_{10} = \underbrace{\sqrt{12 \cdot 7 - 20} + 2}_{10} \quad (w)$$

$$\text{somit: } L = \underline{\underline{\{7\}}}$$

$$f) \quad \sqrt{n} + \sqrt{2n} = 1 - \sqrt{3n} \quad D = \{n \in \mathbf{R} \mid n \geq 0\}$$

$$\sqrt{n} + \sqrt{2}\sqrt{n} + \sqrt{3}\sqrt{n} = 1$$

$$\sqrt{n}(1 + \sqrt{2} + \sqrt{3}) = 1$$

$$\sqrt{n} = \frac{1}{1 + \sqrt{2} + \sqrt{3}}$$

$$|(\)^2$$

$$n = \frac{1}{(1 + \sqrt{2} + \sqrt{3})^2} = \underline{0.0582} \in D$$

$$\text{Kontrolle: } \underbrace{\sqrt{0.0582} + \sqrt{2 \cdot 0.0582}}_{0.5823} = \underbrace{1 - \sqrt{3 \cdot 0.0582}}_{0.5823}$$

$$\text{somit: } L = \underline{\underline{\left\{ \frac{1}{(1 + \sqrt{2} + \sqrt{3})^2} \right\}}}$$

$$342. \text{ d) } \sqrt{a-2} - \sqrt{a-10} = 2\sqrt{a-7}$$

$$a-2 \geq 0 \quad \wedge \quad a-10 \geq 0 \quad \wedge \quad a-7 \geq 0 \quad \rightarrow \quad D = \{a \in \mathbf{R} \mid a \geq 10\}$$

$$\sqrt{a-2} - \sqrt{a-10} = 2\sqrt{a-7} \quad | ()^2$$

$$a-2-2\sqrt{(a-2)(a-10)}+a-10=4(a-7) \quad | \text{TU}$$

$$2a-12-4a+28=2\sqrt{(a-2)(a-10)} \quad | :2$$

$$-a+8=\sqrt{(a-2)(a-10)} \quad | ()^2$$

$$64-16a+\cancel{a^2}=\cancel{a^2}-12a+20 \quad | \text{TU}$$

$$44=4a \quad | :4$$

$$a = \underline{11} \in D$$

$$\text{Kontrolle: } \underbrace{\sqrt{11-2} - \sqrt{11-10}}_2 \neq \underbrace{2\sqrt{11-7}}_4 \quad (\text{f})$$

$$\text{somit: } L = \{ \}$$

$$\text{e) } \sqrt{4m} - 3 = \frac{2m-17}{\sqrt{m}-1} \quad | \cdot (\sqrt{m}-1)$$

$$m \geq 0 \quad \wedge \quad \sqrt{m} \neq 1 \quad \rightarrow \quad D = \{m \in \mathbf{R} \mid m \geq 0 \quad \wedge \quad m \neq 1\}$$

$$(\sqrt{4m}-3) \cdot (\sqrt{m}-1) = 2m-17 \quad | \text{TU}$$

$$\sqrt{4m^2} - \sqrt{4m} - 3\sqrt{m} + 3 = 2m - 17 \quad | \text{TU}$$

$$2m - 2\sqrt{m} - 3\sqrt{m} + 3 = 2m - 17 \quad | \text{Wurzel isolieren}$$

$$-5\sqrt{m} = -20$$

$$\sqrt{m} = 4 \quad | ()^2$$

$$m = \underline{16} \in D$$

$$\text{Kontrolle: } \underbrace{\sqrt{4 \cdot 16} - 3}_5 = \frac{2 \cdot 16 - 17}{\underbrace{\sqrt{16} - 1}_5} \quad (\text{w})$$

$$\text{somit: } L = \{ \underline{16} \}$$

$$f) \frac{5}{\sqrt{u+3}} - \sqrt{u+3} = \sqrt{u-1} \quad | \cdot \sqrt{u+3}$$

$$u > -3 \wedge u \geq 1 \rightarrow D = \{u \in \mathbf{R} | u \geq 1\}$$

$$5 - (u+3) = \sqrt{u-1} \cdot \sqrt{u+3} \quad | \text{Klammer aufl.}$$

$$2 - u = \sqrt{(u-1)(u+3)} \quad | ()^2$$

$$4 - 4u + u^2 = u^2 + 2u - 3 \quad | -u^2 + 4u + 3$$

$$7 = 6u \quad | :6$$

$$u = \frac{7}{6} \in D$$

$$\text{Kontrolle: } \frac{5}{\underbrace{\sqrt{\frac{7}{6}+3}}_{\sqrt{6}}} - \sqrt{\frac{7}{6}+3} = \underbrace{\sqrt{\frac{7}{6}-1}}_{\frac{1}{\sqrt{6}}} \quad (w)$$

$$\text{somit: } L = \left\{ \frac{7}{6} \right\}$$

344. b) $D = \{x \in \mathbf{R} | x \geq 0\}$

$$\sqrt{m+\sqrt{x}} = \sqrt{\frac{m+n}{2}} + \sqrt{\frac{m-n}{2}} = \frac{\sqrt{m+n} + \sqrt{m-n}}{\sqrt{2}} \quad | ()^2$$

$$m + \sqrt{x} = \frac{m+n+2 \cdot \sqrt{(m+n)(m-n)} + m-n}{2} \quad | \text{TU}$$

$$m + \sqrt{x} = \frac{2m+2 \cdot \sqrt{m^2-n^2}}{2} = \frac{\cancel{2}(m+\sqrt{m^2-n^2})}{\cancel{2}} \quad | -m$$

$$\sqrt{x} = \sqrt{m^2-n^2}$$

$$x = \underline{m^2-n^2}$$

Kontrolle: $m=5$ und $n=3 \rightarrow x = 5^2 - 3^2 = 25 - 9 = 16$

$$\sqrt{5+\sqrt{16}} = \sqrt{\frac{5+3}{2}} + \sqrt{\frac{5-3}{2}}$$

$$\underbrace{\sqrt{5+4}}_3 = \underbrace{\sqrt{4+1}}_3 \quad (w)$$

$$L = \left\{ x \in \mathbf{R} | x = m^2 - n^2 \right\}$$

$$345. \text{ a) } \sqrt{2z + \sqrt{z+31}} + \sqrt{2z - \sqrt{z+31}} = 6$$

$$z \geq -2.66 \quad \wedge \quad z \geq 2.91 \quad \rightarrow \quad D = \{z \in \mathbf{R} \mid z \geq 2.91\}$$

mit TI-89 berech.

$$\sqrt{2z + \sqrt{z+31}} + \sqrt{2z - \sqrt{z+31}} = 6$$

 $()^2$

$$2z + \cancel{\sqrt{z+31}} + \underbrace{2\sqrt{2z + \sqrt{z+31}}\sqrt{2z - \sqrt{z+31}}}_{\text{doppeltes Produkt}} + 2z - \cancel{\sqrt{z+31}} = 36 \quad | \text{zusammenfassen}$$

$$4z + 2\sqrt{4z^2 - (z+31)} = 36$$

|Wurzel isolieren

$$2\sqrt{4z^2 - z - 31} = 36 - 4z$$

|:2

$$\sqrt{4z^2 - z - 31} = 18 - 2z$$

 $()^2$

$$\cancel{4z^2} - z - 31 = 324 - 72z + \cancel{4z^2}$$

|+72z + 31

$$71z = 355$$

|:71

$$z = \underline{5} \in D$$

$$\text{Kontrolle: } \sqrt{2 \cdot 5 + \sqrt{5+31}} + \sqrt{2 \cdot 5 - \sqrt{5+31}} = 6$$

$$\underbrace{\sqrt{10 + \sqrt{36}}}_4 + \underbrace{\sqrt{10 - \sqrt{36}}}_2 = 6 \quad (\text{w})$$

$$\text{somit: } L = \underline{\underline{\{5\}}}$$

$$348. \text{ a) } \sqrt{x+\frac{1}{a}} + \sqrt{x-\frac{1}{a}} = \frac{2}{a} \quad D = \left\{ x \in \mathbb{R} \mid x \geq -\frac{1}{a} \wedge x \geq \frac{1}{a} \right\} \wedge a \neq 0$$

$$\sqrt{\frac{ax+1}{a}} + \sqrt{\frac{ax-1}{a}} = \frac{2}{a} \quad |(\)^2$$

$$\frac{ax+1}{a} + 2 \cdot \sqrt{\frac{ax+1}{a}} \cdot \sqrt{\frac{ax-1}{a}} + \frac{ax-1}{a} = \frac{4}{a^2} \quad | \text{Wurzel separieren}$$

$$2 \cdot \sqrt{\frac{ax+1}{a}} \cdot \sqrt{\frac{ax-1}{a}} = \frac{4}{a^2} - \left(\frac{ax+1}{a} + \frac{ax-1}{a} \right) \quad | \text{TU}$$

$$2 \cdot \sqrt{\frac{ax+1}{a}} \cdot \sqrt{\frac{ax-1}{a}} = \frac{4}{a^2} - \frac{2ax}{a} \cdot \frac{a}{a} = \frac{4-2a^2x}{a^2} \quad | :2$$

$$\sqrt{\frac{ax+1}{a}} \cdot \sqrt{\frac{ax-1}{a}} = \frac{2-a^2x}{a^2} \quad |(\)^2$$

$$\frac{ax+1}{a} \cdot \frac{ax-1}{a} = \frac{4-4a^2x+a^4x^2}{a^4} \quad | \text{ausmultiplizieren}$$

$$\frac{a^2x^2-1}{a^2} \cdot \frac{a^2}{a^2} = \frac{4-4a^2x+a^4x^2}{a^4} \quad | \text{erweitern, } \cdot a^4$$

$$\cancel{a^4x^2} - a^2 = 4 - 4a^2x + \cancel{a^4x^2}$$

$$4a^2x = 4 + a^2$$

$$x = \frac{a^2+4}{4a^2}$$

$$\text{Kontrolle: sei } a=2 \rightarrow x = \frac{1}{2}$$

$$\underbrace{\sqrt{\frac{1}{2} + \frac{1}{2}}}_1 + \underbrace{\sqrt{\frac{1}{2} - \frac{1}{2}}}_0 = \underbrace{\frac{2}{2}}_1 \quad (\text{w})$$

$$\text{somit: } L = \left\{ x \mid x = \frac{a^2+4}{4a^2} \right\}$$

$$348. \text{ b) } 4\sqrt{x+2a} - 2\sqrt{x+a} = \frac{4(a-3x)}{\sqrt{4(x+a)}} \quad D = \{x \in \mathbb{R} \mid x \geq -2a \wedge x > -a\} \wedge a \neq 0$$

$$4\sqrt{x+2a} - 2\sqrt{x+a} = \frac{4(a-3x)}{2\sqrt{x+a}} = \frac{2(a-3x)}{\sqrt{x+a}} \quad | \cdot \sqrt{x+a}$$

$$4\sqrt{(x+2a)(x+a)} - 2\sqrt{(x+a)^2} = 2(a-3x) \quad | \text{vereinfachen}$$

$$4\sqrt{(x+2a)(x+a)} - 2(x+a) = 2a - 6x \quad | \text{Wurzel separieren}$$

$$4\sqrt{(x+2a)(x+a)} = 2a - 6x + 2x + 2a = 4a - 4x \quad | : 4$$

$$\sqrt{(x+2a)(x+a)} = a - x \quad | ()^2$$

$$x^2 + ax + 2ax + 2a^2 = a^2 - 2ax \quad | \text{TU}$$

$$5ax = -a^2$$

$$x = -\frac{a^2}{5a} = -\frac{a}{5}$$

$$\text{Kontrolle: sei } a=5 \rightarrow x=-1$$

$$\underbrace{4\sqrt{-1+10}}_{12} - \underbrace{2\sqrt{-1+5}}_4 = \frac{\overbrace{4 \cdot 5 - 12 \cdot (-1)}^{32}}{\underbrace{\sqrt{-4+20}}_4} \quad (\text{w})$$

$$\text{somit: } \underline{\underline{L = \left\{ x \mid x = -\frac{a}{5} \right\}}}$$