Lösungen

**Prüfung: Gleichungen 1. Grades mit einer Unbekannten / Wurzelgl.**

Lösen Sie die folgenden Gleichungen nach x auf und bestimmen Sie die entsprechende Lösungsmenge. G = R

1) $\frac{2a-x}{4a^2-4ab+b^2}=\frac{2-b+x}{4a-2b}$ $\mathbb{D} = \mathbb{R}\ ;\ (2a-b) \neq 0$

$$\frac{2a-x}{4a^2-4ab+b^2} = \frac{2-b+x}{4a-2b}$$

$$\frac{2a-x}{(2a-b)^{\cancel{2}1}} = \frac{2-b+x}{2\cancel{(2a-b)}}$$

$$4a-2x = (2a-b)(2-b+x)$$

$$\cancel{4a}-2x = \cancel{4a}-2ab+2ax-2b+b^2-bx$$

$$-2x+bx-2ax = -2ab-2b+b^2$$

$$x\cancel{(-2a-2+b)} = b\cancel{(-2a-2+b)}$$

$$x = b$$

$$\mathbb{L} = \{b\}$$

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2) $\left[\left(1-p^2\right)\cdot x-1\right]^2+\left(1-2px\right)^2=\left[\left(1+p^2\right)\cdot x+1\right]^2$ $\mathbb{D} = \mathbb{R}$

$$[x-p^2x-1]^2-[x+p^2x+1]^2 = -(1-2px)^2$$

$$(x\cancel{-p^2x}\cancel{-1}+x\cancel{+p^2x}\cancel{+1})(\cancel{x}-p^2x-1\cancel{-x}-p^2x-1) = -(1-2px)^2$$

$$(-1)\cdot \quad 2x(-2p^2x-2) = -(1-4px+4p^2x^2) \quad \cdot(-1)$$

$$\cancel{4p^2x^2}+4x = 1-4px+\cancel{4p^2x^2}$$

$$4x+4px = 1$$

$$x(4+4p = 1$$

$$x = \frac{1}{4+4p}$$

$$\mathbb{L} = \left\{\frac{1}{4+4p}\right\}\ ;\ p \neq -1$$

3) $3\sqrt{x+4}-2\sqrt{x-11}+5\sqrt{x-8} = 0$

$\mathbb{D} = \{x \in \mathbb{R} \mid x \geq 11\}$

$(3 \cdot \sqrt{x+4} + 5\sqrt{x-8})^2 = (2\sqrt{x-11})^2 \quad | \; (\;)^2$

$9(x+4) + 30\sqrt{(x+4)(x-8)} + 25(x-8) = 4\cdot(x-11)$

$9x + 36 + 25x - 200 + 30\sqrt{(\quad} = 4x - 44$

(Quadrieren) $\cancel{30}x - \cancel{120}^{4} = -\cancel{30}^{1}\sqrt{(x+4)(x-8)}$

$(x-4)^2 = (x+4)(x-8)$

$\cancel{x^2} - \cancel{8x} + 16 = \cancel{x^2} - \cancel{8x} + 4x - 32$

$-4x = -48$

$x = 12$

Probe:

$3\sqrt{12+4} - 2\sqrt{12-11} + 5\sqrt{12-8} = 0$

$3\cdot 4 - 2\cdot 1 + 5\cdot 2 = 0$

$12 - 2 + 10 = 0$

$20 = 0 \quad (f)$

$\mathbb{L} = \{\;\}$

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4) $\frac{5(2x^2+3)}{2x+1} - \frac{7x-5}{2x-5} = 5x-6$

$\mathbb{D} = \mathbb{R} \setminus \{-1/2;\ 2{,}5\}$

$\frac{5(2x^2+3)(2x-5) - (7x-5)(2x+1)}{(2x+1)(2x-5)} = 5x-6$

$20x^3 - 50x^2 + 30x - 75 - 14x^2 - 7x + 10x + 5 = (5x-6)(4x^2-8x-5)$

$\cancel{20x^3} - \cancel{64x^2} + 33x - 70 = \cancel{20x^3} - \cancel{40x^2} - 25x$

$-\cancel{24x^2} + 48x + 30$

$33x - 48x + 25x = 30 + 70$

$10x = 100$

$x = 10$

Probe:

$\frac{5(200+3)}{21} - \frac{70-5}{15} = 50-6$

$44 = 44 \quad (w)$

$\mathbb{L} = \{10\}$

5)

$$10-\sqrt{(x-3)(x+13)}=x-1$$

$\mathbb{D} = \{x \in \mathbb{R} \mid x \geq 3 \vee x \leq -13\}$

$$\left(\sqrt{(x-3)(x+13)}\right)^2 = (-x+11)^2 \qquad |\ (\ )^2$$

$$(x-3)(x+13) = x^2 - 22x + 121$$

$$\cancel{x^2} + 13x - 3x - 39 = \cancel{x^2} - 22x + 121$$

$$32x = 160$$

$$x = \frac{160}{32} = 5$$

Probe:

$$10 - \sqrt{(5-3)(5+13)} = 5-1$$

$$10 - \sqrt{2 \cdot 18} = 4$$

$$10 - \sqrt{36} = 4$$

$$10 - 6 = 4$$

$$4 = 4 \quad (w) \qquad \mathbb{L} = \{5\}$$

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6)

Lösen Sie folgende Formel nach $M_2$ auf. Stellen Sie die umgewandelte Formel als gekürzten Bruch dar.

$$w = v\left(1 - \frac{1+k}{1+\frac{M_1}{M_2}}\right)$$

$$w = v\left[1 - \frac{1+k}{\frac{M_2+M_1}{M_2}}\right]$$

$$w = v\left[1 - \frac{M_2(1+k)}{M_1+M_2}\right]$$

$$w = v\left[\frac{(M_1+M_2) - M_2(1+k)}{M_1+M_2}\right]$$

$$w = v\left[\frac{M_1 + \cancel{M_2} - \cancel{M_2} - kM_2}{M_1+M_2}\right]$$

$$w(M_1+M_2) = v(M_1 - kM_2)$$

$$wM_1 + wM_2 = vM_1 - kvM_2$$

$$wM_2 + kvM_2 = vM_1 - wM_1$$

$$M_2(w+kv) = M_1(v-w)$$

$$M_2 = \frac{M_1(v-w)}{w+kv}$$

7)

Lösen Sie die Gleichung nach y auf. Dabei sei $f \neq g$ und $f \neq 0$.

$$\frac{g}{\frac{y}{2f}} - \frac{g-f}{g+f} = \frac{g}{\frac{y}{2f} - \frac{g-f}{g+f} \cdot \frac{y}{2f}}$$

$$\frac{2gf}{y} - \frac{g-f}{g+f} = \frac{g}{\frac{y(g+f) - y(g-f)}{2f(g+f)}}$$

$$\frac{2gf(g+f) - y(g-f)}{y(g+f)} = \frac{g \cdot 2f(g+f)}{y(g+f-g+f)}$$

$$\frac{2g^2f + 2gf^2 - gy + fy}{\cancel{y}(g+f)} = \frac{\cancel{2}\cancel{f}g(g+f)}{\cancel{y} \cdot \cancel{2} \cdot \cancel{f}} = g(g+f)$$

$$2g^2f + 2gf^2 - gy + fy = g(g+f)^2$$

$$-y(g-f) = g(g+f)^2 - 2gf(g+f)$$

$$-y(g-f) = g(g+f)(g-f)$$

$$-y = \frac{g(g+f)\cancel{(g-f)}}{\cancel{(g-f)}}$$

$$y = \underline{\underline{-g(f+g)}}$$

Fz

# Lösungen

**Prüfung: Gleichungen 1. Grades mit einer Unbekannten / Wurzelgl.**

Lösen Sie die folgenden Gleichungen nach x auf und bestimmen Sie die entsprechende Lösungsmenge. G = R

$\mathbb{D} = \mathbb{R} \setminus \{0\}$

1) $$3x - 4\cdot\left(1-\frac{2x}{3}\right) = 15 - 7x\cdot\left(1-\frac{5}{x}\right) + \frac{10}{3}\cdot\left(\frac{5x}{3}+3\right)$$

$$3x - 4 + \frac{8x}{3} = 15 - 7x + \frac{35x}{x} + \frac{50x}{9} + 10$$

$$3x + \frac{8x}{3} + 7x - \frac{50x}{9} = 15 + 10 + 4 + 35$$

$$10x + \frac{8x}{3} - \frac{50x}{9} = 64$$

$$\frac{90x + 24x - 50x}{9} = 64$$

$$\frac{64x}{9} = 64$$

$$x = 9 \qquad \mathbb{L} = \{9\}$$

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2) $$\frac{x+a-b}{a+b} + \frac{x}{2a} - \frac{x-4b}{b} = 1$$

$\mathbb{D} = \mathbb{R}$

$a \neq b \neq 0;\ (a+b) \neq 0$

$$\frac{2ab(x+a-b) + bx(a+b) - 2a(a+b)(x-4b)}{2ab(a+b)} = 1$$

$$\cancel{2abx} + \cancel{2a^2b} - 2ab^2 + abx + b^2x - 2a^2x + 8a^2b - \cancel{2abx} + 8ab^2 = \cancel{2a^2b} + 2ab^2$$

$$abx + 8a^2b + 4ab^2 + b^2x - 2a^2x = 0$$

$$abx + b^2x - 2a^2x = -8a^2b - 4ab^2$$

$$x(-2a^2 + ab + b^2) = 4ab(-2a-b)$$

$$x = \frac{4ab\cancel{(-2a-b)}}{\cancel{(-2a-b)}(a-b)}$$

$$x = \frac{4ab}{a-b}$$

$$\mathbb{L} = \left\{\frac{4ab}{a-b}\right\} \wedge (a-b) \neq 0$$

3) $\dfrac{\frac{x}{5}+1}{\frac{x}{5}-\frac{1}{2}} = \dfrac{x+\frac{5}{3}}{x-\frac{10}{3}}$ $\qquad \mathbb{D} = \mathbb{R} \setminus \{2{,}5;\ \frac{10}{3}\}$

$$\frac{\frac{x+5}{5}}{\frac{2x-5}{10}} = \frac{\frac{3x+5}{3}}{\frac{3x-10}{3}} \quad \Bigg| \quad \frac{x+5}{\cancel{5}} \cdot \frac{\cancel{10}^{\,2}}{2x-5} = \frac{3x+5}{\cancel{3}} \cdot \frac{\cancel{3}}{3x-10}$$

$$2(x+5)(3x-10) = (3x+5)(2x-5)$$
$$2(3x^2-10x+15x-50) = 6x^2-15x+10x-25$$
$$\cancel{6x^2}+10x-100 = \cancel{6x^2}-5x-25$$
$$15x = 75$$
$$x = 5$$

$\mathbb{L} = \{5\}$

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4) $\sqrt{x+2} = \dfrac{1-x}{\sqrt{x-3}} \quad \Big|^2$ $\qquad \mathbb{D} = \{x \in \mathbb{R} \mid x > 3\}$

$$x+2 = \frac{(1-x)^2}{x-3}$$
$$(x+2)(x-3) = 1-2x+x^2$$
$$\cancel{x^2}-x-6 = 1-2x\cancel{+x^2}$$
$$x = 7$$

$\mathbb{L} = \{\ \}$

Probe:

$$\sqrt{7+2} = \frac{1-7}{\sqrt{7-3}}$$
$$\sqrt{9} = \frac{-6}{\sqrt{4}}$$
$$3 = \frac{-6}{2} = -3 \ (f)$$

5) $\sqrt{4x+1}-\sqrt{x+3}=\sqrt{x-2}$ $\qquad \mathbb{D}=\{x\in\mathbb{R} / x\geq 2\}$

$$\left(\sqrt{4x+1}-\sqrt{x+3}\right)^2 = x-2$$
$$4x+1-2\sqrt{(4x+1)(x+3)}+\cancel{x}+3 = \cancel{x}-2$$
$$4x+6 = 2\sqrt{(\ )(\ )}$$
$$\cancel{2}(2x+3) = \cancel{2}\sqrt{(\ )(\ )} \quad |^2$$
$$(2x+3)^2 = (4x+1)(x+3)$$
$$\cancel{4x^2}+\cancel{12x}+9 = \cancel{4x^2}+\cancel{12x}+x+3$$
$$x = 6 \qquad \mathbb{L}=\{6\}$$

Probe:
$$\sqrt{24+1} - \sqrt{6+3} = \sqrt{6-2}$$
$$5 - 3 = 2$$
$$2 = 2 \quad (w)$$

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6) $\dfrac{12{,}5}{4x^2-20x+25}=\dfrac{5x}{4x^2-25}-\dfrac{5}{4x-10}$ $\qquad \mathbb{D}=\mathbb{R}\setminus\{2{,}5;\ -2{,}5\}$

$$\frac{12{,}5}{(2x-5)(2x-5)} = \frac{5x}{(2x-5)(2x+5)} - \frac{5}{2(2x-5)}$$
$$\frac{25(2x+5)}{(2x-5)\cancel{(2x-5)}\cancel{(2x+5)}\cdot\cancel{2}} = \frac{10x-5(2x+5)}{\cancel{2}\cancel{(2x-5)}\cancel{(2x+5)}}$$
$$\frac{50x+125}{2x-5} = \frac{\cancel{10x}-\cancel{10x}-25}{1}$$
$$50x+125 = (2x-5)(-25)$$
$$50x+\cancel{125} = -50x+\cancel{125}$$
$$100x = 0$$
$$x = 0 \qquad \mathbb{L}=\{0\}$$

Probe
$$\frac{12{,}5}{25} = 0 - \frac{5}{-10}$$
$$\frac{1}{2} = \frac{1}{2} \quad (w)$$

Fz

**Prüfung: Gleichungen 1. Grades mit einer Unbekannten / Wurzelgl.**

Lösen Sie die folgenden Gleichungen nach x auf und bestimmen Sie die entsprechende Lösungsmenge. G = R

1) $\frac{11x-1}{2x-1} - \frac{8x-3}{3x-2} = \frac{17x-7}{2(3x-2)}$ $\qquad \mathbb{D} = \mathbb{R} \setminus \{0{,}5 ; \frac{2}{3}\}$

$$\frac{(11x-1)\cdot(3x-2)\cdot 2 - 2(2x-1)(8x-3)}{\cancel{(2x-1)(3x-2)\cdot 2}} = \frac{(17x-7)(2x-1)}{\cancel{2(3x-2)(2x-1)}}$$

$$66x^2 - 44x - 6x + 4 - 32x^2 + 12x + 16x - 6 = 34x^2 - 17x - 14x + 7$$

$$-22x - 2 = -31x + 7$$

$$9x = 9$$

$$x = 1$$

$$\mathbb{L} = \{1\}$$

Probe: $\frac{10}{1} - \frac{5}{1} = \frac{10}{2}$

$5 = 5$ (w)

2) $\frac{1}{3}\left\{\frac{1}{3}\left[\left(\frac{x}{3}-\frac{3}{2}\right)-1.5\right]-1.5\right\}-1.5 = 0$ $\qquad \mathbb{D} = \mathbb{R}$

$$\frac{1}{3}\left\{\frac{1}{3}\left[\frac{2x-9}{6} - 1{,}5\right] - 1{,}5\right\} - 1{,}5 = 0$$

$$\frac{1}{3}\left\{\frac{1}{3}\left[\frac{2x-9-9}{6}\right] - 1{,}5\right\} - 1{,}5 = 0$$

$$\frac{1}{3}\left\{\frac{1}{3}\left[\frac{2x-18}{6}\right] - 1{,}5\right\} - 1{,}5 = 0$$

$$\frac{1}{3}\left\{\frac{2x-18}{18} - 1{,}5\right\} - 1{,}5 = 0$$

$$\frac{1}{3}\left\{\frac{2x-18-27}{18}\right\} - 1{,}5 = 0$$

$$\frac{1}{3}\left\{\frac{2x-45}{18}\right\} - 1{,}5 = 0$$

$$\frac{2x-45-81}{54} = 0$$

$$2x = 126$$

$$x = 63$$

$$\mathbb{L} = \{63\}$$

3. $\left(\sqrt{x+3}-1\right)\cdot\left(\sqrt{x-3}-5\right)=0$

$\mathbb{D}=\{x\in\mathbb{R} \mid x\geq 3\}$

Ein Produkt ist Null, wenn mindestens ein Faktor Null ist!

$$\sqrt{x+3}-1=0$$
$$\sqrt{x+3}=1 \quad |^2$$
$$x+3=1$$
$$x=-2$$

Probe:

$$\left(\sqrt{-2+3}-1\right)\cdot\underbrace{\sqrt{-2-5}}_{\text{nicht def.}}-5=0$$

$$\sqrt{x-3}-5=0$$
$$\sqrt{x-3}=5 \quad |^2$$
$$x-3=25$$
$$x=28$$

Probe

$$\left(\sqrt{28+3}-1\right)\left(\sqrt{28-3}-5\right)=0$$
$$\left(\sqrt{31}-1\right)\left(\sqrt{25}-5\right)=0$$
$$(w) \quad 0=0$$

$$\mathbb{L}=\{28\}$$

4. $\sqrt{3x+\sqrt{7x+7}}=\sqrt{4x+1} \quad |^2$

$\mathbb{D}=\{x\in\mathbb{R} \mid x\geq -1/4\}$

$$3x+\sqrt{7x+7}=4x+1$$
$$\sqrt{7x+7}=(x+1) \quad |^2$$
$$7x+7=(x+1)^2$$
$$7x+7=x^2+2x+1$$
$$0=x^2-5x-6$$
$$0=(x+1)(x-6)$$

$$x_1=-1$$
$$x_2=6$$

$$\mathbb{L}=\{6\}$$

Probe für $x=-1$

$$\sqrt{-3+\sqrt{-7+7}}=\underbrace{\sqrt{-4-1}}_{\text{nicht def.}}$$

für $x=6$

$$\sqrt{18+\sqrt{42+7}}=\sqrt{24+1}$$
$$\sqrt{18+7}=\sqrt{25}$$
$$5=5 \quad (w)$$

5.) $\dfrac{5}{\sqrt{x+3}} - \sqrt{x+3} = \sqrt{x-1}$

$\mathbb{D} = \{x \in \mathbb{R} \mid x \geq 1\}$

$$\frac{5 - \sqrt{x+3} \cdot \sqrt{x+3}}{\sqrt{x+3}} = \sqrt{x-1}$$

$$\frac{5 - x - 3}{\sqrt{x+3}} = \sqrt{x-1}$$

$$\frac{2 - x}{\sqrt{x+3}} = \sqrt{x-1} \quad \Big|^2$$

$$\frac{(2-x)^2}{x+3} = x - 1$$

$$4 - 4x + x^2 = (x-1)(x+3)$$

$$4 - 4x + x^2 = x^2 + 3x - x - 3$$

$$-6x = -7$$

$$x = 7/6$$

$\mathbb{L} = \{7/6\}$

Probe:

$$\frac{5}{\sqrt{7/6+3}} - \sqrt{7/6+3} = \sqrt{7/6-1}$$

$$\frac{5}{\sqrt{\frac{25}{6}}} - \sqrt{\frac{25}{6}} = \sqrt{1/6}$$

$$0{,}408 = 0{,}408 \quad (\text{W})$$

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6.) $\sqrt{2} - \dfrac{\sqrt{2}}{x - \dfrac{\sqrt{2}}{\sqrt{2}-1}} = \dfrac{3}{\sqrt{2}}$

$\mathbb{D} = \mathbb{R} \setminus \left\{ \dfrac{\sqrt{2}}{\sqrt{2}-1} \right\}$

$$\sqrt{2} - \frac{\sqrt{2}}{\frac{x(\sqrt{2}-1) - \sqrt{2}}{\sqrt{2}-1}} = \frac{3}{\sqrt{2}}$$

$$\sqrt{2} - \frac{\sqrt{2}(\sqrt{2}-1)}{x\sqrt{2} - x - \sqrt{2}} = \frac{3}{\sqrt{2}}$$

$$-\frac{2 - \sqrt{2}}{x\sqrt{2} - x - \sqrt{2}} = \frac{3}{\sqrt{2}} - \sqrt{2} = \frac{3 - 2}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$-\frac{2-\sqrt{2}}{x\sqrt{2}-x-\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$-\sqrt{2}\,(2-\sqrt{2}) = x\sqrt{2} - x - \sqrt{2}$$

$$-2\sqrt{2} + 2 = x\sqrt{2} - x - \sqrt{2}$$

$$x - x\sqrt{2} = -\sqrt{2} + 2\sqrt{2} - 2$$

$$x(1-\sqrt{2}) = \sqrt{2} - 2$$

$$x = \frac{\sqrt{2}-2}{1-\sqrt{2}} = \frac{\sqrt{2}\,\cancel{(1-\sqrt{2})}}{\cancel{(1-\sqrt{2})}} \quad \frac{(-1)}{(-1)}$$

$$x = \underline{\sqrt{2}}$$

$$\mathbb{L} = \underline{\underline{\{\sqrt{2}\}}}$$

**Prüfung: Gleichungen 1. Grades mit einer Unbekannten / Wurzelgl.**

Lösen Sie folgenden Gleichungen nach x auf und bestimmen Sie die entsprechende Lösungsmenge. **G = R**

1) $$\frac{\frac{7x}{3}-2}{4} - \frac{\frac{10x-1}{2}}{3} = \frac{\frac{x-8}{4}}{5} - \frac{10}{3}$$

$\underline{D = \mathbb{R}}$

$$\frac{7x-6}{3\cdot 4} - \frac{10x-1}{2\cdot 3} = \frac{x-8}{4\cdot 5} - \frac{10}{3}$$

$$\frac{35x-30-100x+10}{\cancel{60}} = \frac{3x-24-200}{\cancel{60}}$$

$$-65x-20 = 3x-224$$

$$68x = 204$$

$$\underline{x = 3}$$

$$\underline{\underline{\mathbb{L} = \{3\}}}$$

Probe:

$$\frac{\frac{21}{3}-2}{4} - \frac{\frac{30-1}{2}}{3} = \frac{\frac{3-8}{4}}{5} - \frac{10}{3}$$

$$\frac{5}{4} - \frac{14{,}5}{3} = \frac{-5}{20} - \frac{10}{3}$$

$$\frac{75-290}{\cancel{60}} = \frac{-15-200}{\cancel{60}}$$

$$\underline{-215 = -215} \quad (w)$$

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2) $$\frac{3x+3}{x+2} = 5 - \frac{2x-3}{x-1}$$

$\underline{\mathbb{D} = \mathbb{R} \setminus \{1; -2\}}$

$$\frac{3x+3}{x+2} = \frac{5(x-1)-2x+3}{x-1}$$

$$(3x+3)(x-1) = (x+2)(3x-2)$$

$$\cancel{3x^2}+3x-3x-3 = \cancel{3x^2}+6x-2x-4$$

$$4x = 1$$

$$\underline{x = \frac{1}{4}}$$

$$\underline{\mathbb{L} = \left\{\frac{1}{4}\right\}}$$

Probe:

$$\frac{3/4+3}{2{,}25} = 5 - \frac{-2{,}5}{-3/4}$$

$$\underline{1{,}\overline{6} = 1{,}\overline{6}} \quad (w)$$

3) $$\frac{1}{11}\left[\frac{1}{9}\left\{\frac{1}{7}\left[\frac{1}{5}\left(\frac{x-2}{3}-4\right)-6\right]-8\right\}-10\right]=1$$ $\mathbb{D}=\mathbb{R}$

$$\frac{1}{11}\left[\frac{1}{9}\left\{\frac{1}{7}\left[\frac{1}{5}\left(\frac{x-2-12}{3}\right)-6\right]-8\right\}-10\right]=1$$
$$\frac{1}{11}\left[\frac{1}{9}\left\{\frac{1}{7}\left[\frac{x-14-90}{15}\right]-8\right\}-10\right]=1$$
$$\frac{1}{11}\left[\frac{1}{9}\left\{\frac{1}{7}\cdot\frac{x-104}{15}-8\right\}-10\right]=1$$
$$\frac{1}{11}\left[\frac{1}{9}\left\{\frac{x-104-840}{105}\right\}-10\right]=1$$
$$\frac{1}{11}\left[\frac{1}{9}\cdot\frac{x-944}{105}-10\right]=1$$
$$\frac{1}{11}\left[\frac{x-944-9450}{945}\right]=1$$
$$\frac{1}{11}\cdot\frac{x-10394}{945}=1$$
$$x-10394=11\cdot 945$$
$$\underline{x=20'789}$$

Probe:

$$\frac{1}{11}\left[\frac{1}{9}\left\{\frac{1}{7}\left[\frac{1}{5}\left(\frac{20'789-2}{3}-4\right)-6\right]-8\right\}-10\right]=1$$
$$\underline{1=1\ (w)} \qquad \underline{\underline{\mathbb{L}=\{20'789\}}}$$

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Lösen Sie die Formel nach $M_1$ auf.

4) $$M_2=\sqrt{\frac{1+0{,}2\cdot M_1^2}{1{,}4\,M_1^2-0{,}2}}$$

$$M_2^2=\frac{1+0{,}2M_1^2}{1{,}4\,M_1^2-0{,}2} \quad \Big| \text{ quadriert}$$
$$M_2^2(1{,}4\,M_1^2-0{,}2)=1+0{,}2M_1^2$$
$$1{,}4\,M_1^2M_2^2-0{,}2M_2^2=1+0{,}2M_1^2$$
$$1{,}4\,M_1^2M_2^2-0{,}2M_1^2=1+0{,}2M_2^2$$
$$M_1^2(1{,}4M_2^2-0{,}2)=1+0{,}2M_2^2$$

$$M_1^2 = \frac{1+0{,}2\,M_2^2}{1{,}4\,M_2^2 - 0{,}2}$$

$$\underline{\underline{M_1 = \sqrt{\frac{1+0{,}2\,M_2^2}{1{,}4\,M_2^2 - 0{,}2}}}}$$

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5) $\sqrt{x^2-1} = x - 3$

$$\underline{\mathbb{D} = \{x \in \mathbb{R} \mid x \leq -1 \vee x \geq 1\}}$$

$$x^2 - 1 = (x-3)^2$$

$$\cancel{x^2} - 1 = \cancel{x^2} - 6x + 9$$

$$6x = 10$$

$$x = \frac{10}{6} = \underline{\frac{5}{3}}$$

<u>Probe:</u>

$$\sqrt{\frac{25}{9} - 1} = \frac{5}{3} - 3$$

$$\sqrt{\frac{25-9}{9}} = \frac{5-9}{3}$$

$$\sqrt{\frac{16}{9}} = \frac{-4}{3}$$

$$\underline{\frac{4}{3} = \frac{-4}{3}} \quad (f)$$

$$\underline{\underline{\mathbb{L} = \{\ \}}}$$

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6) $3 \cdot \sqrt{x-2} - \sqrt{x+5} = \sqrt{2x+3}$

$$\underline{\mathbb{D} = \{x \in \mathbb{R} \mid x \geq 2\}}$$

$$\left(3\sqrt{x-2} - \sqrt{x+5}\right)^2 = \left(\sqrt{2x+3}\right)^2$$

$$9(x-2) - 2 \cdot 3\sqrt{x-2} \cdot \sqrt{x+5} + (x+5) = 2x+3$$

$$9x - 18 - 6\sqrt{(x-2)(x+5)} + x + 5 = 2x + 3$$

$$8x - 16 = 6\sqrt{(x-2)(x+5)} \quad | :2$$

$$4x - 8 = 3\sqrt{(x-2)(x+5)} \quad |^2$$

$$(4x-8)^2 = 9(x-2)(x+15)$$

$$16x^2 - 64x + 64 = 9x^2 + 27x - 90$$

$$x^2 - 13x + 22 = 0$$

$$\underline{x_1 = 2}$$

$$\underline{x_2 = 11}$$

Probe:

für $x = 2$

$$3\sqrt{0} - \sqrt{7} = \sqrt{7}$$
$$-\sqrt{7} = \sqrt{7} \ (f)$$

für $x = 11$

$$3\sqrt{9} - \sqrt{16} = \sqrt{25}$$
$$9 - 4 = 5$$
$$5 = 5 \ (w)$$

$$\mathbb{L} = \{ 11 \}$$

GSBM

**Prüfung: Gleichungen 1. Grades mit einer Unbekannten / Wurzelgl.**

Lösen Sie folgenden Gleichungen nach x auf und bestimmen Sie die entsprechende Lösungsmenge. **G = R**

1) $$\frac{\frac{7x}{3}-2}{4}-\frac{\frac{10x-1}{2}}{3}=\frac{\frac{x-8}{4}}{5}-\frac{10}{3}$$ $D = \mathbb{R}$

$$\frac{7x-6}{3\cdot 4}-\frac{10x-1}{2\cdot 3}=\frac{x-8}{4\cdot 5}-\frac{10}{3}$$

$$\frac{35x-30-100x+10}{\cancel{60}}=\frac{3x-24-200}{\cancel{60}}$$

$$-65x-20=3x-224$$

$$68x=204$$

$$x=3$$

$\mathbb{L}=\{3\}$

Probe:

$$\frac{\frac{21}{3}-2}{4}-\frac{\frac{30-1}{2}}{3}=\frac{\frac{3-8}{4}}{5}-\frac{10}{3}$$

$$\frac{5}{4}-\frac{14{,}5}{3}=\frac{-5}{20}-\frac{10}{3}$$

$$\frac{75-290}{\cancel{60}}=\frac{-15-200}{\cancel{60}}$$

$$-215=-215 \quad (w)$$

---

2) $$\frac{3x+3}{x+2}=5-\frac{2x-3}{x-1}$$ $D=\mathbb{R}\setminus\{1;-2\}$

$$\frac{3x+3}{x+2}=\frac{5(x-1)-2x+3}{x-1}$$

$$(3x+3)(x-1)=(x+2)(3x-2)$$

$$\cancel{3x^2}+3x-3x-3=\cancel{3x^2}+6x-2x-4$$

$$4x=1$$

$$x=\frac{1}{4}$$

$\mathbb{L}=\left\{\frac{1}{4}\right\}$

Probe:

$$\frac{3/4+3}{2{,}25}=5-\frac{-2{,}5}{-3/4}$$

$$1{,}\overline{6}=1{,}\overline{6} \quad (w)$$

3)

$$\sqrt{x-1+\sqrt{2x+5}}-2=0$$

$$\left(\sqrt{x-1+\sqrt{2x+5}}\right)^2=(2)^2 \quad |\,^2$$

$$x-1+\sqrt{2x+5}=4$$

$$\left(\sqrt{2x+5}\right)^2=(5-x)^2 \quad |\,^2$$

$$2x+5=25-10x+x^2$$

$$0=x^2-12x+20$$

$$\underline{x_1=2}$$

$$\underline{x_2=10}$$

$$\mathbb{L}=\{2\}$$

Probe

für $x=2$

$$\sqrt{2-1+\sqrt{4+5}}-2=0$$

$$\sqrt{2-1+3}-2=0$$

$$2-2=0 \quad (w)$$

für $x=10$

$$\sqrt{10-1+\sqrt{20+5}}-2=0$$

$$\sqrt{9+5}-2=0 \quad (f)$$

4)

Lösen Sie die Formel nach $V_2$ auf.

$$T=\left(\frac{h}{v_1}+\frac{h}{v_2}\right)\frac{b}{s}$$

$$sT=\frac{(hv_2+hv_1)b}{v_1v_2}=\frac{bhv_2+bhv_1}{v_1v_2}$$

$$sTv_1v_2=bhv_2+bhv_1$$

$$sTv_1v_2-bhv_2=bhv_1$$

$$v_2(sTv_1-bh)=bhv_1$$

$$v_2=\frac{bhv_1}{sTv_1-bh}$$

5) $\sqrt{x^2-1} = x-3$  $\mathbb{D} = \{x \in \mathbb{R} \mid x \leq -1 \vee x \geq 1\}$

$$x^2 - 1 = (x-3)^2$$
$$\cancel{x^2} - 1 = \cancel{x^2} - 6x + 9$$
$$6x = 10$$
$$x = \frac{10}{6} = \underline{\frac{5}{3}}$$

Probe:

$$\sqrt{\frac{25}{9} - 1} = \frac{5}{3} - 3$$
$$\sqrt{\frac{25-9}{9}} = \frac{5-9}{3}$$
$$\sqrt{\frac{16}{9}} = \frac{-4}{3}$$
$$\frac{4}{3} = \frac{-4}{3} \quad (f)$$

$\mathbb{L} = \{\ \}$

---

6) $3 \cdot \sqrt{x-2} - \sqrt{x+5} = \sqrt{2x+3}$  $\mathbb{D} = \{x \in \mathbb{R} \mid x \geq 2\}$

$$\left(3\sqrt{x-2} - \sqrt{x+5}\right)^2 = \left(\sqrt{2x+3}\right)^2$$
$$9(x-2) - 2 \cdot 3\sqrt{x-2} \cdot \sqrt{x+5} + (x+5) = 2x+3$$
$$9x - 18 - 6\sqrt{(x-2)(x+5)} + x + 5 = 2x+3$$
$$8x - 16 = 6\sqrt{(x-2)(x+5)} \quad | :2$$
$$4x - 8 = 3\sqrt{(x-2)(x+5)} \quad |^2$$
$$(4x-8)^2 = 9(x-2)(x+5)$$
$$16x^2 - 64x + 64 = 9x^2 + 27x - 90$$
$$x^2 - 13x + 22 = 0$$

$x_1 = 2$

$x_2 = 11$

Probe

für $x = 2$

$$3\sqrt{0} - \sqrt{7} = \sqrt{7}$$
$$-\sqrt{7} = \sqrt{7} \quad (f)$$

für $x = 11$

$$3\sqrt{9} - \sqrt{16} = \sqrt{25}$$
$$3 \cdot 3 - 4 = 5$$
$$5 = 5 \quad (w)$$

$\mathbb{L} = \{11\}$

## Prüfung: Gleichungen 1. Grades mit einer Unbekannten / Wurzelgl.

Lösen Sie folgenden Gleichungen nach x auf und bestimmen Sie die entsprechende Lösungsmenge. G = R

1. $\left(\frac{1}{1+x}-\frac{1}{1-x}\right):\left(\frac{1}{1+\frac{1}{x}}+\frac{1}{1-\frac{1}{x}}\right)=1$ D = $\mathbb{R}\setminus\{0;1;-1\}$

$$\frac{1-x-1-x}{(1+x)(1-x)} : \left(\frac{1}{\frac{x+1}{x}}+\frac{1}{\frac{x-1}{x}}\right)=1$$

$$\frac{-2x}{(1+x)(1-x)} : \left(\frac{x}{x+1}+\frac{x}{x-1}\right)=1$$

$$\frac{-2x}{(1+x)(1-x)} : \frac{x(x-1)+x(x+1)}{(x+1)(x-1)}=1$$

$$\frac{-2x}{(1+x)(1-x)} : \frac{x^2-\cancel{x}+x^2+\cancel{x}}{(x+1)(x-1)}=1$$

$$\frac{-2x}{\cancel{(1+x)}\cancel{(1-x)}} \cdot \frac{\cancel{(x+1)}\cancel{(-x+1)}(-1)}{2x^2}=1$$

$$\frac{\cancel{2}x}{\cancel{2}x^2}=1$$

$$\frac{1}{x}=1$$

$$x=1$$

$\mathbb{L}=\{\ \}$

Bemerkung: $1 \notin \mathbb{D}$

---

2. $\sqrt{x-1+\sqrt{2x+5}}-2=0$ D = $\{x\in\mathbb{R} \mid x \geq -0{,}828\}$

$$\left(\sqrt{x-1+\sqrt{2x+5}}\right)^2=(2)^2 \quad |\ ^2$$

$$x-1+\sqrt{2x+5}=4$$

$$\left(\sqrt{2x+5}\right)^2=(5-x)^2 \quad |\ ^2$$

$$2x+5=25-10x+x^2$$

$$0=x^2-12x+20$$

$$x_1=2$$

$$x_2=10$$

$\mathbb{L}=\{2\}$

Probe

für $x=2$

$$\sqrt{2-1+\sqrt{4+5}}-2=0$$

$$\sqrt{2-1+3}-2=0$$

$$2-2=0 \quad (w)$$

für $x=10$

$$\sqrt{10-1+\sqrt{20+5}}-2=0$$

$$\sqrt{9+5}-2=0 \quad (f)$$

3. $\frac{1}{3}\cdot\left(\frac{1}{3}\cdot\left\{\frac{1}{3}\cdot\left[\frac{1}{3}\cdot\left(\frac{1}{3}\cdot x+2\right)+2\right]+2\right\}+2\right) = 1$ $\quad D = \mathbb{R}$

$$\frac{1}{3}\left(\frac{1}{3}\left\{\frac{1}{3}\left[\frac{1}{3}\left(\frac{x+6}{3}\right)+2\right]+2\right\}+2\right) = 1$$

$$\frac{1}{3}\left(\frac{1}{3}\left\{\frac{1}{3}\left[\frac{x+6+18}{9}\right]+2\right\}+2\right) = 1$$

$$\frac{1}{3}\left(\frac{1}{3}\left\{\frac{x+24+54}{27}\right\}+2\right) = 1$$

$$\frac{1}{3}\left(\frac{x+78+162}{81}\right) = 1$$

$$\frac{x+240}{243} = 1$$

$$x = 243-240$$

$$\underline{x = 3}$$

Probe

$$\frac{1}{3}\left(\frac{1}{3}\left\{\frac{1}{3}\left[\underbrace{\frac{1}{3}\left(\frac{1}{3}\cdot 3+2\right)}+2\right]+2\right\}+2\right) = 1$$

$\mathbb{L} = \{3\}$

$$\frac{1}{3}\left(\frac{1}{3}\left\{\underbrace{\frac{1}{3}\left[1+2\right]+2}\right\}+2\right) = 1$$

$$\frac{1}{3}\left(1+2\right) = 1$$

$$\underline{1 = 1} \quad (w)$$

---

4. $\sqrt{2x+4} - 2\sqrt{2x+2} = \frac{8-24x}{\sqrt{32x+32}}$ $\quad D = \{x \in \mathbb{R} \mid x > -1\}$

$$\sqrt{2x+4} - 2\sqrt{2x+2} = \frac{\cancel{4}(2-6x)}{\cancel{4}\sqrt{2x+2}}$$

$$\sqrt{2x+2}\left(\sqrt{2x+4} - 2\sqrt{2x+2}\right) = 2-6x$$

$$\sqrt{(2x+2)(2x+4)} - 2(2x+2) = 2-6x$$

$$\left(\sqrt{(2x+2)(2x+4)}\right)^2 = (-2x+6)^2 \quad |^2$$

$$(2x+2)(2x+4) = (-2x+6)^2$$

$$\cancel{4x^2} + 12x + 8 = \cancel{4x^2} - 24x + 36$$

$$36x = 28$$

$$\underline{x = 7/9}$$

$\mathbb{L} = \{7/9\}$

Probe $\sqrt{\frac{2\cdot 7}{9}+4} - 2\sqrt{\frac{2\cdot 7}{9}+2} = \frac{8-24\cdot 7/9}{\sqrt{32\frac{\cdot 7}{9}+32}}$

$$\underbrace{\qquad}\qquad$$

$$2{,}357 - 3{,}771 = -1{,}414$$

$$\underline{-1{,}414 = -1{,}414} \ (w)$$

---

5. $\frac{x-\sqrt{a}}{\sqrt{b}}+\frac{x-\sqrt{b}}{\sqrt{a}} = 2$ D = $\mathbb{R}$ $a>0$, $b>0$

$$\frac{\sqrt{a}(x-\sqrt{a})+\sqrt{b}(x-\sqrt{b})}{\sqrt{a}\cdot\sqrt{b}} = 2$$

$$x\sqrt{a} - a + x\sqrt{b} - b = 2\sqrt{a}\cdot\sqrt{b}$$

$$x\sqrt{a} + x\sqrt{b} = 2\sqrt{ab} + a + b$$

$$x(\sqrt{a}+\sqrt{b}) = a + 2\sqrt{ab} + b$$

$$x = \frac{a+2\sqrt{ab}+b}{(\sqrt{a}+\sqrt{b})}$$

$$x = \frac{\cancel{(\sqrt{a}+\sqrt{b})}(\sqrt{a}+\sqrt{b})}{\cancel{(\sqrt{a}+\sqrt{b})}} = \underline{\sqrt{a}+\sqrt{b}}$$

$$\mathbb{L} = \underline{\underline{\{(\sqrt{a}+\sqrt{b})\}}}$$

---

6. Lösen Sie diese Formel nach $V_1$ auf. Stellen Sie $V_1$ als gekürzten Bruch dar.

$$\frac{RTb-2a}{cV_1V_2} = \frac{p_1-p_2}{V_2-V_1}\cdot\left(\frac{b}{c}-\frac{2a}{RTc}\right)$$

$$\frac{RTb-2a}{cV_1V_2} = \frac{p_1-p_2}{V_2-V_1}\left(\frac{bRT-2a}{RTc}\right)$$

$$\cancel{(RTb-2a)}(V_2-V_1)\cdot RTc = (p_1-p_2)\cancel{(bRT-2a)}\,cV_1V_2$$

$$V_2RTc - V_1RTc = p_1cV_1V_2 - p_2cV_1V_2$$

$$-V_1RTc - p_1cV_1V_2 + p_2cV_1V_2 = -V_2RTc \quad |(-1)$$

$$V_1(RTc + p_1cV_2 - p_2cV_2) = V_2RTc$$

$$V_1 = \frac{V_2RT\cancel{c}}{RT\cancel{c}+p_1\cancel{c}V_2-p_2\cancel{c}V_2}$$

$$\underline{\underline{V_1}} = \frac{RTV_2}{RT+V_2(p_1-p_2)}$$

**Prüfung: Gleichungen 1. Grades mit einer Unbekannten / Wurzelgl.**

Lösen Sie folgenden Gleichungen nach x auf und bestimmen Sie die entsprechende Lösungsmenge. G = R

1. $\frac{x+1}{x+3} - \frac{x+4}{x-2} = \frac{1-3x}{x^2+x-6}$ D = $\mathbb{R} \setminus \{2; -3\}$

$$\frac{(x+1)(x-2)-(x+3)(x+4)}{\cancel{(x+3)(x-2)}} = \frac{1-3x}{\cancel{(x+3)(x-2)}}$$

$$\cancel{x^2} + x - 2x - 2 - \cancel{x^2} - 3x - 4x - 12 = 1-3x$$

$$-8x - 14 = 1 - 3x$$

$$-5x = 15$$

$$x = -3$$

$\mathbb{L} = \{ \}$

Bemerkung: $x \notin \mathbb{D}$

2. $\sqrt{x-1+\sqrt{2x+5}} - 2 = 0$ D = $\{x \in \mathbb{R} \mid x \geq -0{,}828\}$

$$\left(\sqrt{x-1+\sqrt{2x+5}}\right)^2 = (2)^2 \quad |\,^2$$

$$x - 1 + \sqrt{2x+5} = 4$$

$$\left(\sqrt{2x+5}\right)^2 = (5-x)^2 \quad |\,^2$$

$$2x+5 = 25 - 10x + x^2$$

$$0 = x^2 - 12x + 20$$

$$x_1 = 2$$

$$x_2 = 10$$

$\mathbb{L} = \{2\}$

Probe

für $x = 2$

$$\sqrt{2-1+\sqrt{4+5}} - 2 = 0$$

$$\sqrt{2-1+3} - 2 = 0$$

$$2 - 2 = 0 \quad (w)$$

für $x = 10$

$$\sqrt{10-1+\sqrt{20+5}} - 2 = 0$$

$$\sqrt{9+5} - 2 = 0 \quad (f)$$

3. Lösen Sie folgende Formel nach $v_2$ auf, und stellen Sie das Resultat in Form eines einzigen gewöhnlichen Bruches dar:

$$T = \left(\frac{h}{v_1} + \frac{h}{v_2}\right)\frac{b}{s}$$

$$sT = \frac{(hv_2 + hv_1)\,b}{v_1 v_2} = \frac{bhv_2 + bhv_1}{v_1 v_2}$$

$$sTv_1v_2 = bhv_2 + bhv_1$$

$$sTv_1v_2 - bhv_2 = bhv_1$$

$$v_2(sTv_1 - bh) = bhv_1$$

$$\underline{\underline{v_2 = \frac{bhv_1}{sTv_1 - bh}}}$$

4. $\sqrt{2x+4} - 2\sqrt{2x+2} = \dfrac{8-24x}{\sqrt{32x+32}}$ $\qquad$ D = $\{x \in \mathbb{R} \mid x > -1\}$

$$\sqrt{2x+4} - 2\sqrt{2x+2} = \frac{\cancel{4}(2-6x)}{\cancel{4}\sqrt{2x+2}}$$

$$\sqrt{2x+2}\left(\sqrt{2x+4} - 2\sqrt{2x+2}\right) = 2-6x$$

$$\sqrt{(2x+2)(2x+4)} - 2(2x+2) = 2-6x$$

$$\left(\sqrt{(2x+2)(2x+4)}\right)^2 = (-2x+6)^2 \quad \Big|\,^2$$

$$(2x+2)(2x+4) = (-2x+6)^2$$

$$\cancel{4x^2} + 12x + 8 = \cancel{4x^2} - 24x + 36$$

$$36x = 28$$

$$\underline{x = 7/9} \qquad \mathbb{L} = \underline{\{7/9\}}$$

5. $\dfrac{1}{\dfrac{x}{x-2}} - \dfrac{1}{\dfrac{x}{x-2} - 2} + \dfrac{4}{\dfrac{x}{x-2} - 3} = 0$ $\qquad$ D = $\mathbb{R} \setminus \{2; 0; 4; 3\}$

$$\frac{x-2}{x} - \frac{1}{\dfrac{x-2x+4}{x-2}} + \frac{4}{\dfrac{x-3x+6}{x-2}} = 0$$

$$\frac{x-2}{x} - \frac{x-2}{-x+4} + \frac{4(x-2)}{-2x+6} = 0$$

$$(x-2)\left[\frac{1}{x} + \frac{1}{x-4} + \frac{2}{-x+3}\right] = 0$$

$$(x-2)\cdot\left[\frac{(x-4)(-x+3)+x(-x+3)+2x(x-4)}{x(x-4)(-x+3)}\right]=0$$

$$(x-2)\left[-\cancel{x^2}+7x-12-\cancel{x^2}+3x+\cancel{2x^2}-8x\right]$$

$$(x-2)(2x-12)=0 \quad |:2$$

$$(x-2)(x-6)=0$$

$$x_1=2 \rightarrow 2\notin \mathbb{D}$$

$$\underline{x_2=6}$$

<u>Probe</u> für $x_2=6$

$$\frac{1}{\frac{6}{4}}-\frac{1}{\frac{6}{4}-2}+\frac{4}{\frac{6}{4}-3}=0$$

$$\frac{2}{3}+\frac{4}{2}-\frac{8}{3}=0 \qquad \mathbb{L}=\{6\}$$

$$\underline{0=0} \quad (w)$$

---

6. $$\frac{\frac{x}{5}+1}{\frac{x}{5}-\frac{1}{2}}=\frac{x+\frac{5}{3}}{x-\frac{10}{3}} \qquad D=\mathbb{R}\setminus\{2{,}5;\ {}^{10}/_{3}\}$$

$$\frac{\frac{x+5}{5}}{\frac{2x-5}{10}}=\frac{\frac{3x+5}{3}}{\frac{3x-10}{3}}$$

$$\frac{\overset{2}{\cancel{10}}(x+5)}{\cancel{5}(2x-5)}=\frac{\cancel{3}(3x+5)}{\cancel{3}(3x-10)}$$

$$2(x+5)(3x-10)=(3x+5)(2x-5)$$

$$\cancel{6x^2}-20x+30x-100=\cancel{6x^2}-15x+10x-25$$

$$15x=75$$

$$\underline{x=5}$$

<u>Probe</u>
$$\frac{\frac{5}{5}+1}{\frac{5}{5}-\frac{1}{2}}=\frac{5+\frac{5}{3}}{5-\frac{10}{3}} \qquad \mathbb{L}=\{5\}$$

$$\frac{2}{\frac{1}{2}}=\frac{\frac{15+5}{3}}{\frac{15-10}{3}}=\frac{20}{\cancel{3}}\cdot\frac{\cancel{3}}{5}=4$$

$$\underline{4=4} \quad (w)$$

Fz