

# Lösungen: Lin. Gl. Systeme / Ungl.

Folgen

A

$$1) \left[ \begin{array}{l} 2x + 3y - 4z = -5 \\ 3x - 5y + 2z = 4 \\ 4x + y - 2z = 5 \end{array} \right] \cdot (-2) \quad \left[ \begin{array}{l} \cdot 3 \\ \cdot (-2) \end{array} \right] +$$

$$\left[ \begin{array}{l} -5y + 6z = 15 \\ 19y - 16z = -23 \end{array} \right] \cdot 19 \quad \left[ \begin{array}{l} \cdot 5 \end{array} \right] +$$

AD = R

$$34z = 170$$

$$z = 5$$

$$y = 3$$

$$x = 3$$

$$\mathbb{L} = \underline{\underline{\{3; 3; 5\}}}$$

$$2) \left[ \begin{array}{l} \frac{4}{2x-y} - \frac{5}{x-2y} = 6 \\ \frac{8}{2x-y} - \frac{7}{x-2y} = 9 \end{array} \right]$$

Substitution

$$a = 2x - y$$

$$b = x - 2y$$

$$\begin{array}{l} I \\ II \end{array} \left[ \begin{array}{l} \frac{4}{a} - \frac{5}{b} = 6 \\ \frac{8}{a} - \frac{7}{b} = 9 \end{array} \right] \cdot (-2) \quad \left[ \begin{array}{l} \cdot (-2) \end{array} \right] +$$

$$\left[ \begin{array}{l} \frac{3}{b} = -3 \end{array} \right] \Rightarrow \begin{array}{l} b = -1 \\ a = 4 \end{array}$$

$$\left[ \begin{array}{l} a = 4 = 2x - y \\ b = -1 = x - 2y \end{array} \right] \cdot (-2) \quad \left[ \begin{array}{l} \cdot (-2) \end{array} \right] +$$

$$6 = 3y \Rightarrow y = 2$$

$$x = 3$$

$$\mathbb{L} = \underline{\underline{\{3; 2\}}}$$

$$3) \left[ \begin{array}{l} \sqrt{x+y} + \sqrt{y+z} = 5 \\ \sqrt{x+y} + \sqrt{x+z} = 7 \\ \sqrt{y+z} + \sqrt{x+z} = 8 \end{array} \right]$$

Substitution

$$\sqrt{x+y} = a$$

$$\sqrt{y+z} = b$$

$$\sqrt{x+z} = c$$

$$\begin{array}{l} \text{I} \\ \text{II} \\ \text{III} \end{array} \left| \begin{array}{l} a + b = 5 \\ a + c = 7 \\ b + c = 8 \end{array} \right| \cdot (-1) \Big] +$$

$$\begin{array}{l} \text{I} + \text{II} \\ \text{III} \end{array} \left| \begin{array}{l} b - c = -2 \\ b + c = 8 \end{array} \right| +$$

$$\underline{2b = 6}$$

$$\begin{aligned} \longrightarrow b &= 3 = \sqrt{y+z} \\ \longrightarrow a &= 2 = \sqrt{x+y} \\ \longrightarrow c &= 5 = \sqrt{x+z} \end{aligned}$$

Quadrieren:

$$\begin{array}{l} \text{I} \\ \text{II} \\ \text{III} \end{array} \left| \begin{array}{l} y + z = 9 \\ x + y = 4 \\ x + z = 25 \end{array} \right| \cdot (-1) \Big] +$$

$$\begin{array}{l} \text{II} + \text{III} \\ \text{I} \end{array} \left| \begin{array}{l} y - z = -21 \\ y + z = 9 \end{array} \right| +$$

$$\begin{aligned} 2y &= -12 \\ y &= -6 \\ x &= 10 \\ z &= 15 \end{aligned}$$

$$\underline{\underline{L = \{10; -6; 15\}}}$$

$$4) \begin{array}{l} \text{I} \\ \text{II} \end{array} \left| \begin{array}{l} \frac{x+y}{xy} = \frac{1}{10} \\ \frac{1}{x} - \frac{1}{y} = 0,05 \end{array} \right| \longrightarrow \begin{array}{l} \text{I} \\ \text{II} \end{array} \left| \begin{array}{l} 10x + 10y = xy \\ y - x = \frac{xy}{20} \end{array} \right|$$

$$x_{\text{I}} = \frac{-10y}{10-y}$$

$$x_{\text{II}} = \frac{20y}{20+y}$$

$$\begin{aligned} x_{\text{I}} &= x_{\text{II}} \\ \frac{-10y}{10-y} &= \frac{20y}{20+y} \end{aligned}$$

$$-10y(20+y) = 20y^2(10-y)$$

$$-20 - y = 20 - 2y$$

$$y = \underline{\underline{40}}$$

$$x_{\text{I}} = \frac{-400}{10-40} = \frac{-400}{-30} = \underline{\underline{\frac{40}{3}}}$$

Die beiden Zahlen sind  $\frac{40}{3}$  und  $\frac{40}{3}$ !

$$5) \frac{x+1}{x+2} > 0$$

$$D = \mathbb{R} \setminus \{-2\}$$

1. Fall

$$\oplus \wedge \oplus \quad x+1 > 0 \rightarrow x > -1$$

$$\oplus \wedge \oplus \quad x+2 > 0 \rightarrow x > -2$$

$$L_1 = \{x \mid x > -1\}$$

2. Fall

$$\ominus \wedge \ominus \quad x+1 < 0 \rightarrow x < -1$$

$$\ominus \wedge \ominus \quad x+2 < 0 \rightarrow x < -2$$

$$L_2 = \{x \mid x < -2\}$$

$$L = \{x \in \mathbb{R} \mid (x > -1) \vee (x < -2)\}$$

$$6) \frac{2x-3}{6x+7} < \frac{1}{3}$$

$$D = \mathbb{N}$$

$$\frac{2x-3}{6x+7} - \frac{1}{3} < 0$$

$$\frac{6x-9-6x-7}{3(6x+7)} < 0$$

$$\frac{-16}{6x+7} < 0$$

$$6x+7 > 0$$

$$x > -7/6$$

$$L = \{x \in \mathbb{N}\}$$

$$L = \mathbb{N}$$

$$7) \frac{x+2}{(x-1)(x+1)} > 0$$

$$D = \mathbb{Q} \setminus \{1; -1\}$$

1. Fall

$$\left. \begin{array}{l} \oplus \wedge \oplus \oplus \\ \oplus \oplus \oplus \end{array} \right\} \begin{array}{l} x+2 > 0 \rightarrow x > -2 \\ x+1 > 0 \rightarrow x > -1 \\ x-1 > 0 \rightarrow x > 1 \end{array} \quad L_1 = \{x \mid x > 1\}$$

$$\left. \begin{array}{l} \oplus \wedge \ominus \ominus \\ \ominus \ominus \ominus \end{array} \right\} \begin{array}{l} x+2 > 0 \rightarrow x > -2 \\ x+1 < 0 \rightarrow x < -1 \\ x-1 < 0 \rightarrow x < +1 \end{array} \quad L_2 = \{x \mid -2 < x < -1\}$$

2. Fall

$$\left. \begin{array}{l} \ominus \wedge \oplus \oplus \\ \ominus \oplus \oplus \end{array} \right\} \begin{array}{l} x+2 < 0 \rightarrow x < -2 \\ x+1 < 0 \rightarrow x < -1 \\ x-1 > 0 \rightarrow x > 1 \end{array} \quad L_3 = \{\}$$

$$\left. \begin{array}{l} \ominus \wedge \oplus \ominus \\ \oplus \oplus \ominus \end{array} \right\} \begin{array}{l} x+2 < 0 \rightarrow x < -2 \\ x+1 > 0 \rightarrow x > -1 \\ x-1 < 0 \rightarrow x < +1 \end{array} \quad L_4 = \{\}$$

$$L = \{x \in \mathbb{Q} \mid (x > 1) \vee (-2 < x < -1)\}$$



Lösungen

$$1) \left| \begin{array}{l} 0,4x + 1,6y - 1,1z = 1,4 \\ 1,6x + 1,9y - 1,7z = 5,6 \\ 1,2x - 2,2y + 0,7z = 2,2 \end{array} \right| \cdot (-4) \cdot (-3)$$

$$\left. \begin{array}{l} \text{I} \quad -1,6x - 6,4y + 4,4z = -5,6 \\ \text{II} \quad 1,6x + 1,9y - 1,7z = 5,6 \end{array} \right\} +$$


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$$\quad \quad \quad -4,5y + 2,7z = 0$$

$$\left. \begin{array}{l} \text{I} \quad -1,2x - 4,8y + 3,3z = -4,2 \\ \text{III} \quad 1,2x - 2,2y + 0,7z = 2,2 \end{array} \right\} +$$


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$$\quad \quad \quad -7y + 4z = -2$$

$$\begin{aligned} -4,5y + 2,7z &= 0 \rightarrow y = \frac{2,7z}{4,5} \\ -7y + 4z &= -2 \rightarrow y = \frac{2 + 4z}{7} \end{aligned}$$

$$\frac{2,7z}{4,5} = \frac{2 + 4z}{7}$$

$$z = \underline{\underline{10}}$$

$$y = \underline{\underline{6}}$$

$$x = \underline{\underline{7}}$$

$$\mathbb{L} = \underline{\underline{\{7; 6; 10\}}}$$

10

$$2) \left\{ \begin{array}{l} \frac{1}{y} + \frac{1}{z} - \frac{1}{x} = 5 \\ \frac{1}{x} + \frac{1}{y} - \frac{1}{z} = 3 \\ \frac{1}{z} - \frac{1}{y} + \frac{1}{x} = 2 \end{array} \right\} + \begin{array}{l} \frac{2}{y} = 8 \\ y = \underline{\underline{\frac{1}{4}}} \end{array}$$

$$\text{Gl. II} + \text{Gl. III} : \frac{2}{x} = 5 \Rightarrow x = \underline{\underline{\frac{2}{5}}}$$

$$\mathbb{L} = \{ \frac{2}{5}; \frac{1}{4}; \frac{2}{7} \}$$

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3) Substitution:

$$\begin{cases} 5(e+f) - \sqrt{3e+f} = 2 \\ 2(e+f) + \sqrt{3e+f} = 5 \end{cases}$$

$$\begin{cases} e+f = x \\ \sqrt{3e+f} = y \end{cases}$$

$$\begin{cases} 5x - y = 2 \\ 2x + y = 5 \end{cases}$$

$$\begin{cases} y_1 = 5x - 2 \\ y_2 = 5 - 2x \end{cases}$$

$$y_1 = y_2 \\ 5x - 2 = 5 - 2x$$

$$7x = 7$$

$$\underline{x = 1}$$

$$\underline{y = 3}$$

$$\begin{array}{l} \text{I} \\ \text{II} \end{array} \begin{cases} e+f = 1 \\ \sqrt{3e+f} = 3 \end{cases} \rightarrow e = 1-f$$

einsetzen in II  $\sqrt{3(1-f)+f} = 3 \quad |^2$

$$3 - 3f + f = 9$$

$$3 - 2f = 9$$

$$-2f = 6$$

$$f = \underline{\underline{-3}} \quad | \quad \underline{\underline{e = 4}}$$

$$\underline{\underline{\Pi = \{4; -3\}}}$$

4)  $s = v \cdot t$

$$\begin{cases} 120 = v_1 \cdot 6\frac{1}{3} - v_2 \cdot 6\frac{1}{3} \\ 120 = v_1 \cdot 5\frac{3}{4} + v_2 \cdot 5\frac{3}{4} \end{cases}$$

$$120 = 6\frac{1}{3}(v_1 - v_2) \quad \rightarrow v_1 = 18,95 + v_2$$

$$120 = 5\frac{3}{4}(v_1 + v_2) \quad \rightarrow v_1 = 20,87 - v_2$$

$$18,95 + v_2 = 20,87 - v_2$$

$$v_2 = \underline{\underline{0,96}} \text{ [km/h]} \quad \text{Flussgeschw.}$$

$$v_1 = \underline{\underline{19,91}} \text{ [km/h]} \quad \text{Schiffgeschw.}$$

AP



$$5) \quad \frac{4+x}{3+2x} > \frac{1}{2} \quad \mathbb{E} = \mathbb{R} \quad | \quad \underline{D = \mathbb{R} \setminus \{-3/2\}}$$

$$\frac{4+x}{3+2x} - \frac{1}{2} > 0 \quad \left| \quad \begin{array}{l} 3+2x > 0 \\ 2x > -3 \\ x > -\frac{3}{2} \end{array} \right. \quad \begin{array}{l} 1/4 \\ 1P \end{array}$$

$$\frac{2(4+x) - 3 - 2x}{2(3+2x)} > 0$$

$$\frac{5}{3+2x} > 0 \quad \left| \quad \mathbb{L} = \{x \mid x > -\frac{3}{2}\} \right. \quad \begin{array}{l} 3/4 \end{array}$$

$$6) \quad \frac{x+1}{x+2} > 0 \quad \underline{D = \mathbb{R} \setminus \{-2\}} \quad 1/4$$

<p>1. Fall</p> <p>⊕ <math>x+1 &gt; 0 \rightarrow x &gt; -1</math></p> <p>⊕ <math>x+2 &gt; 0 \rightarrow x &gt; -2</math></p> <p><math>\mathbb{L}_1 = \{x \mid x &gt; -1\}</math></p>		<p>2. Fall</p> <p>⊖ <math>x+1 &lt; 0 \rightarrow x &lt; -1</math></p> <p>⊖ <math>x+2 &lt; 0 \rightarrow x &lt; -2</math></p> <p><math>\mathbb{L}_2 = \{x \mid x &lt; -2\}</math></p>
<p><math>\mathbb{L} = \{x \mid (x &gt; -1) \vee (x &lt; -2)\}</math></p>		

1P

$$7) \quad \frac{2x+1}{x(x+1)} < 0 \quad \underline{D = \mathbb{R} \setminus \{0; -1\}} \quad 1/4$$

<p>1. Fall</p> <p>⊖ <math>2x+1 &lt; 0 \rightarrow x &lt; -1/2</math></p> <p>⊕ <math>x(x+1) &gt; 0</math></p> <p>⊕ ⊕ <math>\rightarrow x+1 &gt; 0</math></p> <p style="margin-left: 40px;"><math>\rightarrow x &gt; -1</math></p> <p style="margin-left: 40px;"><math>\rightarrow x &gt; 0</math></p> <p>⊖ ⊖</p> <p style="margin-left: 40px;"><math>\rightarrow x+1 &lt; 0 \rightarrow x &lt; -1</math></p> <p style="margin-left: 40px;"><math>\rightarrow x &lt; 0 \rightarrow x &lt; 0</math></p> <p><math>\mathbb{L}_1 = \{x \mid x &lt; -1\}</math></p>		<p>2. Fall</p> <p>⊕ <math>2x+1 &gt; 0 \rightarrow x &gt; -1/2</math></p> <p>⊖ <math>x(x+1) &lt; 0</math></p> <div style="display: flex; justify-content: space-between; align-items: center;"> <div style="width: 60%;"> <p>⊖ <math>x &lt; 0</math></p> <p>⊕ <math>x+1 &gt; 0</math></p> <p style="margin-left: 40px;"><math>\rightarrow x &gt; -1</math></p> </div> <div style="width: 35%; border-left: 1px solid black; padding-left: 5px;"> <p>⊕ <math>x &gt; 0</math></p> <p>⊖ <math>x+1 &lt; 0</math></p> <p style="margin-left: 40px;"><math>\mathbb{L} = \{ \}</math></p> </div> </div> <p><math>\mathbb{L}_2 = \{x \mid -1/2 &lt; x &lt; 0\}</math></p>
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1P

$$\mathbb{L} = \{x \mid (x < -1) \vee (-1/2 < x < 0)\}$$



Lösen Sie die folgenden Aufgaben ohne Rechner!

1. 
$$\begin{array}{l} \text{I} \\ \text{II} \end{array} \left| \begin{array}{l} (x-8)(y-5) = (x-9)(y-4) \\ (x-8)(y-5) = y(x-9) - 4 \end{array} \right|$$

$$\begin{array}{l} \text{I} \\ \text{II} \end{array} \left| \begin{array}{l} xy - 5x - 8y + 40 = xy - 4x - 9y + 36 \\ \quad \quad \quad -x + y = -4 \\ xy - 5x - 8y + 40 = xy - 9y - 4 \\ \quad \quad \quad -5x + y = -44 \end{array} \right|$$

$$\begin{array}{l} \text{I} \\ \text{II} \end{array} \left| \begin{array}{l} -x + y = -4 \\ -5x + y = -44 \end{array} \right| \cdot (-1) \left. \vphantom{\begin{array}{l} \text{I} \\ \text{II} \end{array}} \right\} +$$

$$4x \quad | = 40$$

$$x = 10$$

$$y = 6$$

Probe

$$\text{I} \quad 2 \cdot 1 = 1 \cdot 2 \quad (\text{w})$$

$$\text{II} \quad 2 \cdot 1 = 6 \cdot 1 - 4$$

$$2 = 2 \quad (\text{w})$$

$$\underline{\underline{\mathbb{L} = \{10; 6\}}}$$

2. 
$$\left| \begin{array}{l} x+4y=2z+p \\ y-3x=z+2p \\ 2x+z=3(y+p) \end{array} \right|$$

$$\left| \begin{array}{l} x + 4y = p \\ -3x + y - z = 2p \\ 2x - 3y + z = 3p \end{array} \right| \cdot (-2) \left. \vphantom{\begin{array}{l} x + 4y = p \\ -3x + y - z = 2p \\ 2x - 3y + z = 3p \end{array}} \right\} +$$

$$\left| \begin{array}{l} -x - 2y - z = 5p \\ 7x + 2y = -3p \end{array} \right| \left. \vphantom{\begin{array}{l} -x - 2y - z = 5p \\ 7x + 2y = -3p \end{array}} \right\} +$$

$$\underline{\underline{\mathbb{L} = \left\{ \frac{p}{3}; \frac{-8p}{3}; \frac{-17p}{3} \right\}}}$$

$$\begin{array}{l} 6x \quad | = 2p \\ x = \frac{p}{3} \end{array}$$

$$y = -\frac{8}{3}p$$

$$z = -\frac{17p}{3}$$



3. Lösen Sie mit einer geeigneten Substitution!

TBM

$$\begin{cases} \frac{7u}{u-v} + 3u + 3v = 75 \\ \frac{9u}{u-v} - 4u - 4v = 10 \end{cases}$$

Subst.  $\frac{u}{u-v} = x$   
 $u+v = y$

$$\begin{array}{l} \text{I} \\ \text{II} \end{array} \left| \begin{array}{l} 7x + 3y = 75 \\ 9x - 4y = 10 \end{array} \right| \begin{array}{l} \cdot 4 \\ \cdot 3 \end{array} \Bigg] +$$

$$55x \quad / \quad = \quad 330$$

$$x = 6$$

$$y = 11$$

Re subst

Gl. I  $\frac{u}{u+v} = 6$

II  $u+v = 11$

$$\Rightarrow \begin{array}{l} u = 6 \\ v = 5 \end{array}$$

$$\underline{\underline{L = \{6; 5\}}}$$

GSBM

$$\begin{cases} \frac{3}{a+b} = 5 - \frac{4}{a-b} \\ -\frac{9}{a+b} = -1 - \frac{2}{a-b} \end{cases}$$

Subst.  $a+b = x$   
 $a-b = y$

$$\begin{array}{l} \text{I} \\ \text{II} \end{array} \left| \begin{array}{l} \frac{3}{x} = 5 - \frac{4}{y} \\ -\frac{9}{x} = -1 - \frac{2}{y} \end{array} \right| \begin{array}{l} \cdot 3 \\ \cdot 3 \end{array}$$

$$0 = 14 - \frac{14}{y}$$

$$14y = 14$$

$$y = 1$$

$$x = 3$$

Resubst.

$$\left. \begin{array}{l} a+b = 3 \\ a-b = 1 \end{array} \right\} +$$

$$2a \quad / \quad = \quad 4$$

$$a = 2$$

$$b = 1$$

$$\underline{\underline{L = \{2; 1\}}}$$

4. Die Summe und der Quotient zweier Zahlen ergeben je 5. Bestimmen Sie die beiden Zahlen.

$$\begin{array}{l} \text{I} \\ \text{II} \end{array} \left| \begin{array}{l} x + y = 5 \\ \frac{x}{y} = 5 \end{array} \right| \begin{array}{l} \rightarrow x = 5 - y \\ \rightarrow x = 5y \end{array}$$

gleichsetzen:

$$5 - y = 5y$$

$$6y = 5$$

$$y = \frac{5}{6}$$

$$\Rightarrow x = 5 - \frac{5}{6} = \frac{25}{6}$$

$$\underline{\underline{L = \left\{ \frac{25}{6}; \frac{5}{6} \right\}}}$$



5.  $\frac{x+5}{x-2} \geq 0$

$G = \mathbb{R}$

$D = ?$

$D = \mathbb{R} \setminus \{2\}$

1. Fall

$$\left. \begin{array}{l} \oplus \quad x+5 \geq 0 \Rightarrow x \geq -5 \\ \oplus \quad x-2 > 0 \Rightarrow x > 2 \end{array} \right\} \underline{M_1 = \{x \in \mathbb{R} \mid x > 2\}}$$

2. Fall

$$\left. \begin{array}{l} \ominus \quad x+5 \leq 0 \Rightarrow x \leq -5 \\ \ominus \quad x-2 < 0 \Rightarrow x < 2 \end{array} \right\} \underline{M_2 = \{x \in \mathbb{R} \mid x \leq -5\}}$$

$M = \{x \in \mathbb{R} \mid x > 2 \vee x \leq -5\}$

6.  $|-3x+2| < 5$

$G = \mathbb{R}$

$D = \mathbb{R}$

1. Fall

$$\begin{array}{l} -3x+2 < 5 \\ -3x < 3 \quad (-1) \\ 3x > -3 \\ \underline{x > -1} \end{array}$$

2. Fall

$$\begin{array}{l} -3x+2 > -5 \\ -3x > -7 \quad (-1) \\ 3x < 7 \\ \underline{x < \frac{7}{3}} \end{array}$$

$M = \{x \in \mathbb{R} \mid -1 < x < \frac{7}{3}\}$

7.  $|2-x| \cdot |1+x| = 2$

$G = \mathbb{R}$

$D = \mathbb{R}$

1. Fall  $\oplus \cdot \oplus$

$$\begin{array}{l|l} 2-x > 0 & 1+x > 0 \\ \underline{x < 2} & \underline{x > -1} \end{array}$$

$$\begin{aligned} (2-x)(1+x) &= 2 \\ 2+x-x^2 &= 2 \\ -x^2+x &= 0 \quad (-1) \\ x^2-x &= 0 \\ x(x-1) &= 0 \end{aligned}$$

$x_1 = 0$

$x_2 = 1$

2. Fall  $\ominus \cdot \ominus$

$$\begin{array}{l|l} 2-x < 0 & 1+x < 0 \\ \underline{x > 2} & \underline{x < -1} \end{array}$$

nicht mögl.!

3. Fall

⊕ · ⊖

$$\begin{array}{l|l} 2-x > 0 & 1+x < 0 \\ \hline x < 2 & x < -1 \end{array}$$

$$(2-x)(1+x) = -2$$

$$-x^2 + x + 2 = -2$$

$$-x^2 + x + 4 = 0$$

$$\underline{x_1 = -1,561}$$

$$\underline{x_2 = 2,5615}$$

4. Fall

⊖ · ⊕

$$\begin{array}{l|l} 2-x < 0 & 1+x > 0 \\ \hline x > 2 & x > -1 \end{array}$$

$$(2-x)(1+x) = -2$$

$$\underline{x_1 = -1,561}$$

$$\underline{x_2 = 2,5615}$$

$$\underline{\underline{\mathbb{L} = \{0; 1; -1,561; 2,5615\}}}$$



1.

$$\left| \begin{array}{r} 5x + 3y - z = 10 \\ x - 2y + 2z = 12 \\ -2x + 4y + z = -19 \end{array} \right| \cdot 2 \quad ] +$$

$$\left| \begin{array}{r} 11x + 4y \quad | = 32 \\ 3x + 7y \quad | = -9 \end{array} \right| \cdot 3 \quad ] +$$

$$\left| \begin{array}{r} 3x + 7y \quad | = -9 \\ \cdot (-11) \end{array} \right| +$$

$$-65y = 195$$

$$y = -3$$

$$x = 4$$

$$z = 1$$

$$\underline{\underline{L = \{4; -3; 1\}}}$$

2.

$$\left| \begin{array}{r} 4x - \frac{5y}{6} = \left(-\frac{2}{3}\right) \\ \frac{2x}{5} + 3y = 32 \end{array} \right| \Rightarrow$$

$$\frac{24x - 5y}{6} = \frac{-4}{6}$$

$$\frac{2x + 15y}{5} = \frac{160}{5}$$

$$\left| \begin{array}{r} I \quad 24x - 5y = -4 \\ II \quad 2x + 15y = 160 \end{array} \right| \cdot 3 \quad ] +$$

$$74x = 148$$

$$x = 2$$

$$y = 10,4$$

$$\underline{\underline{L = \{2; 10,4\}}}$$

3. Lösen Sie mit einer geeigneten Substitution !

TBM

$$\left| \begin{array}{r} \frac{2}{x + \frac{1}{y}} = \frac{5}{2x - \frac{1}{y}} \\ \frac{4}{x + \frac{1}{y}} = \frac{10}{\frac{1}{-2x} - \frac{1}{y}} + 1 \end{array} \right|$$

GSBM

$$\left| \begin{array}{r} \frac{8}{z+1} - \frac{1}{y+z-2} = 7 \\ \frac{13}{z+1} + \frac{2}{y+z-2} = 15 \end{array} \right|$$

Subst.

$$a = \frac{2}{x + \frac{1}{y}}$$

$$b = \frac{5}{2x - \frac{1}{y}}$$

$$a = z + 1$$

$$b = y + z - 2$$



$$\begin{array}{l} \text{I} \\ \text{II} \end{array} \left| \begin{array}{l} a = b \\ 2a = -2b + 1 \end{array} \right| \cdot (-2) \quad +$$

$$0 = -4b + 1$$

$$b = \frac{1}{4} ; a = \frac{1}{4}$$

Rücksubst

$$\text{I} \quad \left| \frac{1}{4} = \frac{2}{x + \frac{1}{y}} \right| \rightarrow x + \frac{1}{y} = 8$$

$$\text{II} \quad \left| \frac{1}{4} = \frac{5}{2x - \frac{1}{y}} \right| \rightarrow 2x - \frac{1}{y} = 20$$

$$3x - \frac{1}{y} = 28$$

$$x = \frac{28}{3}$$

$$y = -\frac{3}{4}$$

$$\mathbb{L} = \left\{ \frac{28}{3} ; -\frac{3}{4} \right\}$$

$$\begin{array}{l} \text{I} \\ \text{II} \end{array} \left| \begin{array}{l} \frac{8}{a} - \frac{1}{b} = 7 \\ \frac{13}{a} + \frac{2}{b} = 15 \end{array} \right|$$

$$\frac{29}{a} - \frac{1}{b} = 29$$

$$a = 1$$

$$b = 1$$

Rücksubst

$$1 = z + 1$$

$$1 = y + z - 2 \quad (-)$$

$$0 = -y + 3$$

$$y = 3$$

$$z = 0$$

$$\mathbb{L} = \{3; 0\}$$

4.  $1 - 3(x - 4) \geq 2(5 - x)$   $G = \mathbb{R}$   $D = \mathbb{R}$

$$1 - 3x + 12 \geq 10 - 2x$$

$$-x \geq -3 \quad | \cdot (-1)$$

$$x \leq 3$$

$$\mathbb{L} = \{x \in \mathbb{R} \mid x \leq 3\}$$

5.  $\frac{x+52}{x+2} < 11$   $G = \mathbb{R}$   $D = \mathbb{R} \setminus \{-2\}$

$$\frac{x+52}{x+2} - 11 < 0$$

$$\frac{x+52-11x-22}{x+2} < 0$$

$$x+2$$

$$\frac{-10x+30}{x+2} < 0 \quad | :10$$

$$x+2$$

$$\frac{-x+3}{x+2} < 0 \quad | \cdot (-1)$$

$$x+2$$

$$\frac{x-3}{x+2} > 0$$

1. Fall

$$\left. \begin{array}{l} \oplus \quad x - 3 > 0 \Rightarrow x > 3 \\ \oplus \quad x + 2 > 0 \Rightarrow x > -2 \end{array} \right\} L_1 = \{x \in \mathbb{R} \mid x > 3\}$$

2. Fall

$$\left. \begin{array}{l} \ominus \quad x - 3 < 0 \Rightarrow x < 3 \\ \ominus \quad x + 2 < 0 \Rightarrow x < -2 \end{array} \right\} L_2 = \{x \in \mathbb{R} \mid x < -2\}$$

$$\underline{\underline{L = \{x \in \mathbb{R} \mid x < -2 \vee x > 3\}}}$$

6.

$$|2x - 6| = 4$$

$G = \mathbb{R}$

$D = \mathbb{R}$

1. Fall

$$\begin{aligned} 2x - 6 &= 4 \\ 2x &= 10 \\ \underline{x} &= \underline{5} \end{aligned}$$

2. Fall

$$\begin{aligned} 2x - 6 &= -4 \\ 2x &= 2 \\ \underline{x} &= \underline{1} \end{aligned}$$

Probe

$$\begin{aligned} |10 - 6| &= 4 \\ \underline{4} &= \underline{4} \quad (\text{w}) \end{aligned}$$

$$\begin{aligned} |2 - 6| &= 4 \\ |-4| &= 4 \\ \underline{4} &= \underline{4} \quad (\text{w}) \end{aligned}$$

$$\underline{\underline{L = \{5; 1\}}}$$

7.

$$\frac{7y+3}{2} - \frac{9y+3}{3} \geq \frac{2(7y+3)}{4}$$

$G = \mathbb{R}$

$D = \mathbb{R}$

$$\frac{6(7y+3)}{12} - \frac{4(9y+3)}{12} - \frac{6(7y+3)}{12} \geq 0 \quad | \cdot 12$$

$$42y + 18 - 36y - 12 - 42y - 18 \geq 0$$

$$-36y - 12 \geq 0 \quad | : 12$$

$$-3y \geq 1$$

$$\underline{\underline{y \leq -\frac{1}{3}}}$$

$$\underline{\underline{L = \{y \in \mathbb{R} \mid y \leq -\frac{1}{3}\}}}$$



**BM-Math. Prüfung: Lineare Gleichungssysteme / Ungleichungen**  
Gleichungen und Ungleichungen mit Beträgen

Lösen Sie die folgenden Aufgaben ohne Rechner!

TBM

GSBM

1. a) 
$$\begin{cases} 0,4x + 1,6y - 1,1z = 1,4 \\ 1,6x + 1,9y - 1,7z = 5,6 \\ 1,2x - 2,2y + 0,7z = 2,2 \end{cases}$$

b) 
$$\begin{cases} x + y + z = 9 \\ 2x + 5z = 24 \\ 5x + 10y - z = 36 \end{cases}$$

1. a) TBM

$$\begin{array}{l} \text{I)} \\ \text{a)} \end{array} \left| \begin{array}{l} 0,4x + 1,6y - 1,1z = 1,4 \\ 1,6x + 1,9y - 1,7z = 5,6 \\ 1,2x - 2,2y + 0,7z = 2,2 \end{array} \right| \cdot (-4) \cdot (-3)$$

$$\begin{array}{l} \text{I} \\ \text{II} \end{array} \left. \begin{array}{l} -1,6x - 6,4y + 4,4z = -5,6 \\ 1,6x + 1,9y - 1,7z = 5,6 \end{array} \right\} +$$


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$$-4,5y + 2,7z = 0$$

$$\begin{array}{l} \text{I} \\ \text{III} \end{array} \left. \begin{array}{l} -1,2x - 4,8y + 3,3z = -4,2 \\ 1,2x - 2,2y + 0,7z = 2,2 \end{array} \right\} +$$


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$$-7y + 4z = -2$$

$$\begin{array}{l} -4,5y + 2,7z = 0 \rightarrow y = \frac{2,7z}{4,5} \\ -7y + 4z = -2 \rightarrow y = \frac{2 + 4z}{7} \end{array}$$

$$\frac{2,7z}{4,5} = \frac{2 + 4z}{7}$$

$$z = \underline{\underline{10}}$$

$$y = \underline{\underline{6}}$$

$$x = \underline{\underline{7}}$$

$$K = \underline{\underline{\{7; 6; 10\}}}$$



1b) GSBM

$$\begin{array}{l}
 \left[ \begin{array}{l}
 x + y + z = 9 \\
 2x \quad \quad + 5z = 24 \\
 5x + 10y - z = 36
 \end{array} \right. \left. \begin{array}{l}
 \cdot (+10) \\
 \\
 (-1)
 \end{array} \right\} + \\
 \hline
 \left[ \begin{array}{l}
 5x \quad \quad - 11z = 54 \\
 2x \quad \quad + 5z = 24
 \end{array} \right. \left. \begin{array}{l}
 \cdot 2 \\
 \cdot (-5)
 \end{array} \right\} + \\
 \hline
 \begin{array}{l}
 -3z = -12 \\
 \underline{z = 4}
 \end{array} \quad \Rightarrow \quad \underline{x = 2} \quad ; \quad \underline{y = 3} \\
 \\
 \underline{\underline{L = \{2; 3; 4\}}}
 \end{array}$$

Probe:

$$\begin{array}{l}
 \text{I} \quad \left| \begin{array}{l} 2 + 3 + 4 = 9 \end{array} \right| (w) \\
 \text{II} \quad \left| \begin{array}{l} 4 \quad \quad + 20 = 24 \end{array} \right| (w) \\
 \text{III} \quad \left| \begin{array}{l} 10 + 30 - 4 = 36 \end{array} \right| (w)
 \end{array}$$

2. Lösen Sie mit einer geeigneten Substitution!

TBM

a)

$$\left| \begin{array}{l}
 5e + 5f - \sqrt{3e+f} = 2 \\
 2e + 2f + \sqrt{3e+f} = 5
 \end{array} \right|$$

GSBM

b)

$$\left| \begin{array}{l}
 \frac{4}{2x-y} - \frac{5}{x-2y} = 6 \\
 \frac{8}{2x-y} - \frac{7}{x-2y} = 9
 \end{array} \right|$$

Za)

Substitution:

$$\left| \begin{array}{l}
 5(e+f) - \sqrt{3e+f} = 2 \\
 2(e+f) + \sqrt{3e+f} = 5
 \end{array} \right| \quad \left| \begin{array}{l}
 e+f = x \\
 \sqrt{3e+f} = y
 \end{array} \right.$$

$$\left| \begin{array}{l}
 5x - y = 2 \\
 2x + y = 5
 \end{array} \right| \quad \begin{array}{l}
 y_1 = 5x - 2 \\
 y_2 = 5 - 2x
 \end{array}$$

TBM

$$\begin{array}{l}
 y_1 = y_2 \\
 5x - 2 = 5 - 2x \\
 7x = 7 \\
 \underline{x = 1}
 \end{array}$$

$$\underline{y = 3}$$

$$\begin{array}{l|l} \text{I} & e + f = 1 \quad \rightarrow e = 1 - f \\ \text{II} & \sqrt{3e + f} = 3 \end{array}$$

einsetzen in II  $\sqrt{3(1-f) + f} = 3 \quad |^2$

$$3 - 3f + f = 9$$

$$3 - 2f = 9$$

$$-2f = 6$$

$$f = \underline{\underline{-3}} \quad / \quad \underline{\underline{e = 4}}$$

$$\underline{\underline{L = \{4; -3\}}}$$

$$26) \quad \underline{\text{GSBM}} \quad \left| \begin{array}{r} \frac{4}{2x-y} - \frac{5}{x-2y} = 6 \\ \frac{8}{2x-y} - \frac{7}{x-2y} = 9 \end{array} \right.$$

Substitution

$$a = 2x - y$$

$$b = x - 2y$$

$$\begin{array}{l} \text{I} \quad \frac{4}{a} - \frac{5}{b} = 6 \\ \text{II} \quad \frac{8}{a} - \frac{7}{b} = 9 \end{array} \quad \left. \begin{array}{l} \cdot (-2) \\ \cdot 7 \end{array} \right\} + \quad \left. \begin{array}{l} \cdot 7 \\ \cdot (-5) \end{array} \right\}$$

$$/ \quad \frac{3}{b} = -3 \quad \Rightarrow \quad b = -1$$

$$-\frac{12}{a} \quad / = -3 \quad \Rightarrow \quad a = 4$$

$$\begin{array}{l} a = 4 = 2x - y \\ b = -1 = x - 2y \end{array} \quad \left| \cdot (-2) \right.$$

$$6 = 3y$$

$$y = \underline{\underline{2}}$$

$$x = \underline{\underline{3}}$$

$$\underline{\underline{L = \{3; 2\}}}$$

Probe:

$$\begin{array}{r} \frac{4}{6-2} - \frac{5}{3-4} = 6 \\ \frac{4}{4} - \frac{5}{-1} = 6 \\ 1 + 5 = 6 \quad (w) \\ \hline \frac{8}{4} - \frac{7}{-1} = 9 \quad (w) \end{array}$$

$$3) \left\{ \begin{array}{l} \frac{1}{y} + \frac{1}{z} - \frac{1}{x} = 5 \\ \frac{1}{x} + \frac{1}{y} - \frac{1}{z} = 3 \\ \frac{1}{z} - \frac{1}{y} + \frac{1}{x} = 2 \end{array} \right\} + \frac{2}{y} = 8$$

$$y = \frac{1}{4}$$

$$\text{Gl. II} + \text{Gl. III} : \frac{2}{x} = 5 \Rightarrow x = \frac{2}{5}$$

$$\mathbb{L} = \left\{ \frac{2}{5}; \frac{1}{4}; \frac{2}{7} \right\}$$

$$4) \left| \frac{3x+1}{2x-2} \right| = 1$$

$$\mathbb{D} = \mathbb{R} \setminus \{1\}$$

1. Fall

$$\frac{3x+1}{2x-2} = 1$$

$$3x+1 = 2x-2$$

$$x = -3$$

2. Fall

$$\frac{3x+1}{2x-2} = -1$$

$$3x+1 = -2x+2$$

$$5x = 1$$

$$x = \frac{1}{5}$$

Probe

$$\left| \frac{-9+1}{-6-2} \right| = 1$$

$$\left| \frac{-8}{-8} \right| = 1$$

$$1 = 1 \quad (\omega)$$

$$\left| \frac{\frac{3}{5}+1}{\frac{2}{5}-2} \right| = 1$$

$$\left| \frac{\frac{8}{5}}{-\frac{8}{5}} \right| = 1$$

$$|-1| = 1$$

$$1 = 1 \quad (\omega)$$

$$\mathbb{L} = \left\{ -3; \frac{1}{5} \right\}$$



$$5) \frac{4+x}{3+2x} > \frac{1}{2}$$

$$E = \mathbb{R} \quad | \quad D = \mathbb{R} \setminus \{-\frac{3}{2}\}$$

$$\frac{4+x}{3+2x} - \frac{1}{2} > 0$$

$$\frac{2(4+x) - 3 - 2x}{2(3+2x)} > 0$$

$$\frac{5}{3+2x} > 0$$

$$3+2x > 0$$

$$2x > -3$$

$$x > \frac{-3}{2}$$

$$\mathbb{L} = \left\{ x \in \mathbb{R} \mid x > -\frac{3}{2} \right\}$$

$$6) \frac{x+1}{x+2} > 0$$

$$D = \mathbb{R} \setminus \{-2\}$$

1. Fall

$$\oplus \quad x+1 > 0 \rightarrow x > -1$$

$$\oplus \quad x+2 > 0 \rightarrow x > -2$$

$$\mathbb{L}_1 = \{x \mid x > -1\}$$

2. Fall

$$\ominus \quad x+1 < 0 \rightarrow x < -1$$

$$\ominus \quad x+2 < 0 \rightarrow x < -2$$

$$\mathbb{L}_2 = \{x \mid x < -2\}$$

$$\mathbb{L} = \left\{ x \in \mathbb{R} \mid (x > -1) \vee (x < -2) \right\}$$

$$7) \frac{2x+1}{x(x+1)} < 0$$

$$D = \mathbb{R} \setminus \{0; -1\}$$

1. Fall

$$\ominus \quad 2x+1 < 0 \rightarrow x < -\frac{1}{2}$$

$$\oplus \quad x(x+1) > 0$$

$$\oplus \cdot \oplus \rightarrow x+1 > 0$$

$$\rightarrow x > -1$$

$$\rightarrow x > 0$$

$$\ominus \cdot \ominus$$

$$\rightarrow x+1 < 0 \rightarrow x < -1$$

$$\rightarrow x < 0 \rightarrow x < 0$$

$$\mathbb{L}_1 = \{x \mid x < -1\}$$

2. Fall

$$\oplus \quad 2x+1 > 0 \rightarrow x > -\frac{1}{2}$$

$$\ominus \quad x(x+1) < 0$$

$$\ominus \quad x < 0$$

$$\oplus \quad x+1 > 0$$

$$\rightarrow x > -1$$

$$\oplus \quad x > 0$$

$$\ominus \quad x+1 < 0$$

$$\mathbb{L} = \{ \}$$

$$\mathbb{L}_2 = \{x \mid -\frac{1}{2} < x < 0\}$$

$$\mathbb{L} = \left\{ x \in \mathbb{R} \mid (x < -1) \vee \left(-\frac{1}{2} < x < 0\right) \right\}$$