5 Rationale Zahlen

5.12 Übungen Frommenwiler

55. b) $\frac{12m}{-4} - \frac{8n}{-4} + \frac{2}{-4} = \underline{\underline{-3m + 2n - \frac{1}{2}}}$

d) $\frac{a^2}{a^2} - \frac{a}{a^2} - \frac{1}{a^2} = \underline{\underline{1 - \frac{1}{a} - \frac{1}{a^2}}}$

e) $\frac{8m^2}{-4m^2 n} + \frac{12mn}{-4m^2 n} - \frac{4n^2}{-4m^2 n} - \frac{(4mn)(4mn)}{-4m^2 n} = \underline{\underline{-\frac{2}{n} - \frac{3}{m} + \frac{n}{m^2} + 4n}}$

56. a) $\frac{-x^3 + 6x}{7} = \underline{\underline{-\frac{1}{7}x^3 + \frac{6}{7}x}}$

b) $\frac{2a^2 - 4a + 6}{4} = \underline{\underline{\frac{1}{2}a^2 - a + \frac{3}{2}}}$

d) $\frac{x - \sqrt{3}x + 1.5}{3} = \frac{x\left(1 - \sqrt{3}\right) + 1.5}{3} = \underline{\underline{\left(\frac{1 - \sqrt{3}}{3}\right)x + \frac{1}{2}}}$

57. a) $(a - b)\left(\frac{b + a}{ab}\right) = \underline{\underline{\frac{a^2 - b^2}{ab}}}$

b) $\left(\frac{m^2 - n^2}{mn}\right)\left(\frac{m^2 + n^2}{mn}\right) = \underline{\underline{\frac{m^4 - n^4}{(mn)^2}}}$

c) $$\left[\frac{a+1}{a(a+1)}-\frac{a}{a(a+1)}\right]^2=\left[\frac{a+1-a}{a(a+1)}\right]^2=\left[\frac{1}{a(a+1)}\right]^2=\underline{\underline{\frac{1}{a^2(a+1)^2}}}$$

d)
$$\frac{\overset{1}{\cancel{x}}}{x^2+y^2}\cdot\frac{2x^2-(x-y)(x+y)}{\underset{1}{\cancel{x}}(x+y)}=\frac{2x^2-(x^2-y^2)}{(x^2+y^2)(x+y)}=$$
$$\frac{2x^2-x^2+y^2}{(x^2+y^2)(x+y)}=\frac{\overset{1}{\cancel{x^2+y^2}}}{\underset{1}{\cancel{(x^2+y^2)}}(x+y)}=\underline{\underline{\frac{1}{x+y}}}$$

58. a)
$$\frac{u}{u+1}\left(\frac{1}{u}-u\cdot\frac{u}{u}\right)-\frac{1}{1-u}\left(\frac{1}{u}+u\cdot\frac{u}{u}\right)=\frac{\overset{1}{\cancel{u}}}{u+1}\cdot\frac{\overbrace{1-u^2}^{\text{3. Binom}}}{\underset{1}{\cancel{u}}}-\frac{\overset{1}{\cancel{u}}}{1-u}\cdot\frac{1+u^2}{\underset{1}{\cancel{u}}}=$$
$$\frac{(1-u)\overset{1}{\cancel{(1+u)}}}{\underset{1}{\cancel{u+1}}}-\frac{1+u^2}{1-u}=(1-u)\cdot\frac{1-u}{1-u}-\frac{1+u^2}{1-u}=\frac{1-2u+u^2-1-u^2}{1-u}=\underline{\underline{\frac{-2u}{1-u}}}=\underline{\underline{\frac{2u}{u-1}}}$$

b)
$$\frac{a(a+b)-b(a-b)}{(a-b)(a+b)}\cdot\frac{b^2-a^2}{a^2b^2}+\frac{1}{b^2}=\frac{a^2+ab-ab+b^2}{(a-b)\underset{1}{\cancel{(a+b)}}}\cdot\frac{(b-a)\overset{1}{\cancel{(b+a)}}}{a^2b^2}+\frac{1}{b^2}=$$
$$\frac{a^2+b^2}{\underset{1}{\cancel{(a-b)}}}\cdot\frac{-\overset{1}{\cancel{(-b+a)}}}{a^2b^2}+\frac{1}{b^2}=\frac{-a^2-b^2}{a^2b^2}+\frac{a^2}{a^2b^2}=\frac{-\overset{1}{\cancel{b^2}}}{a^2\underset{1}{\cancel{b^2}}}=\underline{\underline{\frac{-1}{a^2}}}=\underline{\underline{-\frac{1}{a^2}}}$$

c)
$$(a-1)\cdot\frac{a-a^2+1}{1-a}\cdot\frac{\overbrace{(a+1)^2-a^2}^{\text{3. Binom}}}{a+1}=$$
$$(a-1)\cdot\frac{(a-a^2+1)(-1)}{(1-a)(-1)}\cdot\frac{\overbrace{(a+1+a)(a+1-a)}^{\text{3. Binom angewendet}}}{a+1}=$$
$$\cancel{(a-1)}\cdot\frac{-a+a^2-1}{\cancel{a-1}}\cdot\frac{2a+1}{a+1}=\underline{\underline{\frac{(a^2-a-1)(2a+1)}{a-1}}}$$

d)
$$\frac{1}{x-1}\cdot\frac{(x-1)(x+1)}{(1-x)(1+x)}\cdot\frac{(1-x)^2}{(1+x)^2}\cdot\frac{x^2(x-1)-(x-1)}{(x-1)^2}=$$

$$\frac{(1-x)^2\overbrace{(x^2-1)}^{\text{Binom}}(x-1)}{(1-x)(1+x)^2(x-1)^2}=\frac{(1-x)(x-1)(x+1)}{(1+x)^2(x-1)}=\underline{\underline{\frac{1-x}{1+x}}}$$

e)
$$x\left(\frac{x^2-3x+2}{x^2}\right)^2\left[\left(1-\frac{x}{x-2}\right)\left(1+\frac{x}{x-2}\right)\right]=\frac{x(x^2-3x+2)^2}{x^4}\cdot\frac{x-2-x}{x-2}\cdot\frac{x-2+x}{x-2}=$$

$$\frac{\left[(x-1)(x-2)\right]^2}{x^3}\cdot\frac{-2}{x-2}\cdot\frac{2x-2}{x-2}=\frac{(x-1)^2(x-2)^2(-2)\cdot2(x-1)}{x^3(x-2)^2}=$$

$$\underline{\underline{-\frac{4(x-1)^3}{x^3}}}=\underbrace{-\frac{4\left[-(-x+1)\right]^3}{x^3}=-\frac{4\left[(-1)^3(1-x)^3\right]}{x^3}}_{\text{Minuszeichen kann in den Term }(x-1)^3\text{ gebracht werden}}=\underline{\underline{\frac{4(1-x)^3}{x^3}}}$$

59. a)
$$\frac{\frac{2}{3}\cdot15}{\frac{4}{5}\cdot15}=\frac{10}{12}=\underline{\underline{\frac{5}{6}}}$$

b)
$$\frac{\frac{2}{a}\cdot a}{b\cdot a}=\underline{\underline{\frac{2}{ab}}}$$

c)
$$\frac{x\cdot y}{\frac{3}{y}\cdot y}=\underline{\underline{\frac{xy}{3}}}$$

d)
$$\frac{\frac{a}{a-1}}{a+1}\cdot\frac{a-1}{a-1}=\underline{\underline{\frac{a}{a^2-1}}}$$

e)
$$\frac{\frac{2n}{3}}{n-2}\cdot\frac{n-2}{n-2}=\underline{\underline{\frac{2n^2-4n}{3}}}=\underline{\underline{\frac{2n(n-2)}{3}}}$$

f) $\dfrac{\frac{a}{x}\cdot x^3}{\frac{b}{x^3}\cdot x^3} = \underline{\underline{\dfrac{ax^2}{b}}}$

60 a) $\dfrac{\left(\frac{3}{4}-\frac{1}{3}\right)\cdot 12}{\left(\frac{1}{2}+\frac{1}{3}-\frac{1}{4}\right)\cdot 12} = \dfrac{9-4}{6+4-3} = \underline{\underline{\dfrac{5}{7}}}$

b) $\dfrac{\left(1+\frac{1}{a}\right)\cdot a}{\left(a+\frac{1}{a}\right)\cdot a} = \underline{\underline{\dfrac{a+1}{a^2+1}}}$

c) $\dfrac{\left(\frac{1}{m}-\frac{1}{n}\right)\cdot mn}{(m-n)\cdot mn} = \dfrac{(n-m)(-1)}{(m-n)mn(-1)} = \dfrac{\overset{1}{\cancel{(n-m)}}\,(-1)}{\underset{1}{\cancel{(-m+n)}}\,mn} = \underline{\underline{-\dfrac{1}{mn}}}$

e) $\dfrac{\left(\frac{ac}{bd}\right)\cdot bd}{\left(\frac{a}{b}-\frac{a+b}{d}\right)\cdot bd} = \dfrac{ac}{ad-(a+b)b} = \underline{\underline{\dfrac{ac}{ad-ab-b^2}}}$

f) $\dfrac{\left(1-\frac{1}{x}\right)\cdot x^3}{\left(\frac{1}{x^2}+\frac{1}{x^3}\right)\cdot x^3} = \underline{\underline{\dfrac{x^3-x^2}{x+1}}}$

61. d) $\dfrac{\left(\frac{a}{b}+1\right)\cdot ab}{\left(\frac{b}{a}+1\right)\cdot ab} = \dfrac{a^2+ab}{b^2+ab} = \dfrac{a\,\overset{1}{\cancel{(a+b)}}}{b\,\underset{1}{\cancel{(b+a)}}} = \underline{\underline{\dfrac{a}{b}}}$

e) $$\frac{\left(\frac{4}{a^2}-\frac{4}{ab}+\frac{1}{b^2}\right)\cdot 2a^2b^2}{\left(\frac{1}{a}-\frac{1}{2b}\right)\cdot 2a^2b^2}=\frac{8b^2-8ab+2a^2}{2ab^2-a^2b}=$$

$$\frac{2\overbrace{\left(4b^2-4ab+a^2\right)}^{\text{Binom}}}{ab\left(2b-a\right)}=\frac{2\left(2b-a\right)^{\not 2}}{ab\,\underbrace{\left(2b-a\right)}_{1}}=\underline{\underline{\frac{2\left(2b-a\right)}{ab}}}$$

f) $$\frac{\left(\frac{1}{a-1}+1\right)\cdot(a-1)}{\left(\frac{a}{a-1}-1\right)\cdot(a-1)}=\frac{1+a-1}{a-(a-1)}=\frac{a}{1}=\underline{\underline{a}}$$

62 a) $$\frac{\left(u+\frac{1}{u-1}\right)\cdot(u-1)}{\left(u-\frac{1}{u-1}\right)\cdot(u-1)}=\underline{\underline{\frac{u^2-u+1}{u^2-u-1}}}$$

b) $$\frac{\left(\frac{a}{1-a}+1\right)\cdot(1+a)(1-a)}{\left(\frac{1}{1+a}-1\right)\cdot(1+a)(1-a)}=\frac{a+a^2+1-a^2}{1-a-\left(1-a^2\right)}=\frac{a+1}{1-a-1+a^2}=\frac{a+1}{a^2-a}=\underline{\underline{\frac{a+1}{a(a-1)}}}$$

c) $$\frac{\left(x-y-\frac{x-y}{x+y}\right)\cdot(y)(x+y)}{\left(\frac{x}{y}-\frac{x}{x+y}\right)\cdot(y)(x+y)}=\frac{(x-y)(y)(x+y)-(x-y)(y)}{x(x+y)-xy}=$$

$$\frac{(x-y)(y)(x+y-1)}{x(x+y-y)}=\underline{\underline{\frac{y(x-y)(x+y-1)}{x^2}}}$$

d) $$\frac{\left[\frac{b}{(b-1)(b+1)}\right]\cdot(b-1)(b+1)}{\left(\frac{1}{b+1}-\frac{1}{b-1}\right)\cdot(b-1)(b+1)}=\frac{b}{b-1-(b+1)}=\frac{b}{b-1-b-1}=\frac{b}{-2}=-\underline{\underline{\frac{b}{2}}}$$

e) $$\frac{\frac{a}{a+b}+\frac{b}{a-b}}{\frac{1}{a+b}-\frac{1}{b-a}\cdot\frac{(-1)}{(-1)}}=\frac{\left(\frac{a}{a+b}+\frac{b}{a-b}\right)\cdot(a+b)(a-b)}{\left(\frac{1}{a+b}-\frac{-1}{a-b}\right)\cdot(a+b)(a-b)}=\frac{a(a-b)+b(a+b)}{a-b-(-a-b)}=$$

$$\frac{a^2-ab+ab+b^2}{a-b+a+b}=\underline{\underline{\frac{a^2+b^2}{2a}}}$$

f) $$\frac{\left[\frac{(x+1)^2}{(x-1)(x+1)}\right]\cdot(x+1)(x-1)(x-1)}{\left[\frac{1}{x+1}-\frac{(1-x)(1+x)}{(x-1)(x-1)}\right]\cdot(x+1)(x-1)(x-1)}=\frac{(x+1)^2(x-1)}{(x-1)(x-1)-(1-x)(1+x)(x+1)}=$$

$$\frac{(x+1)^2(x-1)}{(x-1)(x-1)-[-(x-1)](1+x)(x+1)}=\frac{(x+1)^2\,\cancel{(x-1)}}{\cancel{(x-1)}\,[x-1-[-1](1+x)(x+1)]}=$$

$$\frac{(x+1)^2}{x-1-(-1)(x^2+2x+1)}=\frac{(x+1)^2}{x-1-(-x^2-2x-1)}=\frac{(x+1)^2}{x-1+x^2+2x+1}=\underline{\underline{\frac{(x+1)^2}{x(x+3)}}}$$

63. c) $$\frac{u-\frac{1}{u}}{u-\frac{u\cdot(u)}{\left(u+\frac{1}{u}\right)\cdot(u)}}=\frac{\left(u-\frac{1}{u}\right)\cdot(u^2+1)\cdot(u)}{\left[u-\frac{u^2}{(u^2+1)}\right]\cdot(u^2+1)\cdot(u)}=$$

$$\frac{u^2(u^2+1)-(u^2+1)}{u^2(u^2+1)-u^3}=\frac{(u^2+1)(u^2-1)}{u^2\left[(u^2+1)-u\right]}=\underline{\underline{\frac{u^4-1}{u^2(u^2-u+1)}}}$$

d) $$\frac{a}{a+\frac{a\cdot(a-x)}{\left(1-\frac{a}{a-x}\right)\cdot(a-x)}}=\frac{a}{a+\frac{a^2-ax}{(a-x-a)}}=\frac{a\cdot(-x)}{\left(a+\frac{a^2-ax}{-x}\right)\cdot(-x)}=$$

$$\frac{-ax}{-ax+a^2-ax}=\frac{-ax}{a^2-2ax}=\frac{-\cancel{a}^{1}x}{\cancel{a}_{1}(a-2x)}=\underline{\underline{\frac{-x}{a-2x}}}=\underline{\underline{\frac{x}{2x-a}}}$$

64. a)

$$\frac{\frac{e^2-25}{e}}{\frac{-(e^2-2e-15)}{\frac{(e-3)(e+3)}{-(e-3)}}-2e}=\frac{\frac{(e-5)(e+5)}{e}}{\frac{(e-5)\cancel{(e+3)}\cancel{(e-3)}}{\cancel{(e-3)}\cancel{(e+3)}}-2e}=$$

$$\frac{\frac{(e-5)(e+5)}{e}}{e-5-2e}=\frac{(e-5)(e+5)(-1)}{e(-e-5)(-1)}=\frac{(-e+5)\cancel{(e+5)}}{e\cancel{(e+5)}}=\underline{\underline{\frac{5-e}{e}}}$$

b)

$$\frac{\frac{\frac{1}{a^2-1}}{a}}{1-\frac{1}{1-\frac{1}{1-\frac{1}{a}}}}=\frac{\frac{a}{(a-1)(a+1)}}{1-\frac{1}{1-\frac{a}{a-1}}}=\frac{\frac{a}{(a-1)(a+1)}}{1-\frac{a-1}{a-1-a}}=$$

$$\frac{\frac{a}{(a-1)(a+1)}}{1-\frac{(a-1)(-1)}{(-1)(-1)}}=\frac{\frac{a}{(a-1)(a+1)}}{1-(-a+1)}=\frac{\cancel{a}^{1}}{(a-1)(a+1)\cancel{a}_{1}}=\underline{\underline{\frac{1}{a^2-1}}}$$

c)

$$\frac{\frac{2b-a}{ab}}{2a+\frac{(2b-3a)^3}{\frac{\cancel{3}(9a^2+4b^2)\cancel{(3a+2b)}}{\cancel{3}\cancel{(3a+2b)}}-12ab}}=\frac{\frac{2b-a}{ab}}{2a+\frac{(2b-3a)^3}{\underbrace{9a^2-12ab+4b^2}}}=\frac{\frac{2b-a}{ab}}{2a+\frac{(2b-3a)^3}{(2b-3a)^2}}=$$

Reihenfolge ändern (einfacher): $4b^2-12ab+9a^2$

$$\frac{\frac{2b-a}{ab}}{2a+(2b-3a)}=\frac{\cancel{2b-a}}{ab\cancel{(2b-a)}}=\underline{\underline{\frac{1}{ab}}}$$